Aristotle's Classification of Number in *Metaphysics M* 6, 1080a15–37

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AT the beginning of *Metaphysics M* 6, Aristotle decides to examine the views of those who think that numbers are separate substances and the causes of existing things.[1] The rest of the chapter falls into two parts: a theoretical account of the different kinds of numbers that can be conceived (1080a15–b11), followed by a historical survey of the views of Aristotle's predecessors concerning the nature of number (1080b11–33), which ends with the contention that all the views outlined are impossible (1080b33–36).

The classification Aristotle puts forward in 1080a15–37 betrays misunderstanding of the concept of number and also of Plato's ideal numbers or ideas of numbers. Aristotle refers to this doctrine as that of ἀσύμβλητοι ἀριθμοί, that is, incomparable or, even better, inassociable numbers.[2] These numbers, however, are *not* congeries of units, as Aristotle thinks they are, but merely the hypostatization of the universals which constitute the series of natural numbers.[3] These points must be made at the outset in order to clarify that it is only because he considers number to be a congeries of abstract monads that Aristotle offers the following theoretical, *a priori* classification of number according to the nature of the units (1080a15–37):

[1] This is the third question announced in the first chapter of *M*, *cf.* 1076a29–32. The thinkers referred to are the Pythagoreans and the Platonists, *cf.* W. D. Ross, *Aristotle's Metaphysics* II (repr. with corrections, Oxford 1953) 426–27.

[2] As L. Robin says (*La théorie platonicienne des idées et des nombres d'après Aristote* [Paris 1908, repr. Hildesheim 1963] 272 n.1), inassociable numbers (*i.e.* numbers which cannot be added, subtracted, multiplied or divided) is a more appropriate translation of ἀσύμβλητοι ἀριθμοί. Following the usage of most English scholars, however, I refer to them as 'incomparable numbers'.

[3] *Cf.* J. Cook Wilson, *CR* 18 (1904) 247–60; Ross, *op.cit.* (*supra* n.1) 427; H. F. Cherniss, *Aristotle's Criticism of Plato and the Academy* I (Baltimore 1944) 513–24 and *The Riddle of the Early Academy* (Berkeley 1945) 33–37. In connection with Aristotle's misunderstanding of Plato's concept of number as such, there is no need to distinguish between the ideal numbers of Plato's 'earlier theory' and the idea-numbers of the 'later theory' Aristotle ascribes to him. *Cf.* Wilson, *op.cit.* esp. 249–51 and 253–55; Cherniss, *Aristotle's Criticism* 513–16.

ἀνάγκη δ', εἴπερ ἐϲτὶν ὁ ἀριθμὸϲ φύϲιϲ τιϲ καὶ μὴ ἄλλη τίϲ ἐϲτιν αὐτοῦ ἡ οὐϲία ἀλλὰ τοῦτ' αὐτό, ὥϲπερ φαϲί τινεϲ, ἤτοι εἶναι τὸ μὲν πρῶτόν τι αὐτοῦ τὸ δ' ἐχόμενον, ἕτερον ὂν τῷ εἴδει ἕκαϲτον—καὶ τοῦτο ἢ ἐπὶ τῶν μονάδων εὐθὺϲ ὑπάρχει καὶ ἔϲτιν ἀϲύμβλητοϲ ὁποιαοῦν μονὰϲ ὁποιᾳοῦν μονάδι, ἢ εὐθὺϲ ἐφεξῆϲ πᾶϲαι καὶ ϲυμβληταὶ ὁποιαιοῦν ὁποιαιϲοῦν, οἷον λέγουϲιν εἶναι τὸν μαθηματικὸν ἀριθμόν (ἐν γὰρ τῷ μαθηματικῷ οὐδὲν διαφέρει οὐδεμία μονὰϲ ἑτέρα ἑτέραϲ)· ἢ τὰϲ μὲν ϲυμβλητὰϲ τὰϲ δὲ μή (οἷον εἰ ἔϲτι μετὰ τὸ ἓν πρώτη ἡ δυάϲ, ἔπειτα ἡ τριὰϲ καὶ οὕτω δὴ ὁ ἄλλοϲ ἀριθμόϲ, εἰϲὶ δὲ ϲυμβληταὶ αἱ ἐν ἑκάϲτῳ ἀριθμῷ μονάδεϲ, οἷον αἱ ἐν τῇ δυάδι τῇ πρώτῃ αὑταῖϲ, καὶ αἱ ἐν τῇ τριάδι τῇ πρώτῃ αὑταῖϲ, καὶ οὕτω δὴ ἐπὶ τῶν ἄλλων ἀριθμῶν· αἱ δ' ἐν τῇ δυάδι αὐτῇ πρὸϲ τὰϲ ἐν τῇ τριάδι αὐτῇ ἀϲύμβλητοι, ὁμοίωϲ δὲ καὶ ἐπὶ τῶν ἄλλων τῶν ἐφεξῆϲ ἀριθμῶν· διὸ καὶ ὁ μὲν μαθηματικὸϲ ἀριθμεῖται μετὰ τὸ ἓν δύο, πρὸϲ τῷ ἔμπροϲθεν ἑνὶ ἄλλο ἕν, καὶ τὰ τρία πρὸϲ τοῖϲ δυϲὶ τούτοιϲ ἄλλο ἕν, καὶ ὁ λοιπὸϲ δὲ ὡϲαύτωϲ· οὗτοϲ δὲ μετὰ τὸ ἓν δύο ἕτερα ἄνευ τοῦ ἑνὸϲ τοῦ πρώτου, καὶ ἡ τριὰϲ ἄνευ τῆϲ δυάδοϲ, ὁμοίωϲ δὲ καὶ ὁ ἄλλοϲ ἀριθμόϲ)· ἢ τὸν μὲν εἶναι τῶν ἀριθμῶν οἷοϲ ὁ πρῶτοϲ ἐλέχθη, τὸν δ' οἷον οἱ μαθηματικοὶ λέγουϲι, τρίτον δὲ τὸν ῥηθέντα τελευταῖον.

This is Ross's text. His explanation of lines 17–23 must be accepted, for it is clear from the context that, though syntactically *ἢ τὰϲ μὲν κτλ.* in line 23 is coordinate with *ἤτοι εἶναι κτλ.* in line 17, in sense it is coordinate with *ἢ ἐπὶ τῶν μονάδων κτλ.* in line 18 and with *ἢ εὐθὺϲ ἐφεξῆϲ κτλ.* in line 20.[4] But Ross goes astray in the classification of number he infers from this whole passage.

[4] The syntax could be normalized by emending *τὰϲ μὲν ϲυμβλητὰϲ τὰϲ δὲ μή* (line 23) to *αἱ μὲν ϲυμβληταὶ αἱ δὲ μή*; but, in view of the absence of any variant, it seems preferable to keep the reading of the MSS. as *lectio difficilior*. Be that as it may, there can be no question that the hypothesis of line 23 is in sense coordinate with those of lines 18 and 20, since it is clear that the kind of number described in lines 23–30 and 33–35 is such that *τὸ μὲν πρῶτόν τι αὐτοῦ τὸ δ' ἐχόμενον, ἕτερον ὂν τῷ εἴδει ἕκαϲτον*. This consideration suffices to refute the interpretation of A. Schwegler (*Die Metaphysik des Aristoteles* IV [Tübingen 1848] 311–12), who takes the hypothesis of line 23 as coordinate with that of line 17. (Schwegler's interpretation of 1080a35–37, which he adopts from the ps.-Alexander, simply cannot be got out from the text. *Cf.* Ross, *op.cit.* (*supra* n.1) 426–27.) For similar reasons, I cannot accept the suggestion of J. Annas (*Aristotle's Metaphysics, Books M and N* [Oxford 1976] 163–64) to excise *ἢ* in line 18. For, if we do excise it, the *ἢ* of line 23 would introduce a different possibility from that introduced by *ἤτοι* in line 17. This would be awkward, however, since the

According to 1080a35–37 (but taking into account also what is said in 1080a17–35) there can be three kinds of numbers: (a) incomparable numbers with units all incomparable, (b) mathematical number with units all comparable, and (c) incomparable numbers with the units of each number comparable with each other but incomparable with those of other numbers. The problem is that after introducing incomparable numbers in 1080a17–18 Aristotle goes into the nature of the units themselves, so that in 1080a15–35 he appears to be offering the following classification: (a) incomparable numbers (i) with the units all incomparable, (ii) with the units all comparable, and (iii) with the units of each number comparable with each other but incomparable with those of other numbers. But Ross is mistaken, I think, in inferring from 1080a15–37 the following classification: on the one hand, the belief in either (a, i) or (a, ii) or (a, iii), and, on the other hand, the belief in all three kinds of numbers. This interpretation causes him to contend that Aristotle has omitted the belief in three different combinations of numbers and that he has confused incomparable numbers the units of which are all comparable (a, ii) with mathematical number (b) the units of which must necessarily be all comparable.[5]

It would be more than remarkable, however, if Aristotle were guilty of the confusion Ross ascribes to him, since in the very next chapter of *Metaphysics M* he states that if all the units are comparable and undifferentiated there is only mathematical number,[6] whereas if all the units are incomparable this number cannot be mathematical number.[7] And it is implicit even in 1080a15–35 that numbers such as (a, ii) cannot be incomparable, since all the monads are said to be *συμβληταί*. We must notice, moreover, that in 1080a21 Aristotle says

number described in lines 23–30 and 33–35 would then have to be different from that described in lines 17–20 (and not merely from that of lines 18–20, as it is if we keep the text of the MSS.); so that, apart from saying that the two numbers differ in the nature of their respective units, the emended text would seem to imply that the number of lines 23–30 and 33–35 is not *τὸ μὲν πρῶτόν τι αὐτοῦ τὸ δ' ἐχόμενον, ἕτερον ὂν τῷ εἴδει ἕκαστον* (lines 17–18), which it is. Surely Aristotle would have repeated this phrase in lines 23ff had he not written the *ᾗ* of line 18. The mere fact that from line 18 onward Aristotle discusses the monads is an indication that the three kinds of numbers described in lines 18–35 are divisions of the class established in lines 17–18. (It should be added that Annas adopts Ross's interpretation of 1080a35–37, against which *cf.* my remarks in the text *infra.*)

[5] *Cf.* Ross, *op.cit.* (*supra* n.1) 426. His note on 1080b10–11 is also wrong.

[6] *Cf. Metaph.* 1081a5–7.

[7] *Cf. Metaph.* 1081a17–21.

οἷον λέγουσιν εἶναι τὸν μαθηματικὸν ἀριθμόν (*i.e.* the very number which is not and cannot be incomparable) and that (a, ii) could not be such that *τὸ μὲν πρῶτόν τι αὐτοῦ τὸ δ' ἐχόμενον, ἕτερον ὂν τῷ εἴδει ἕκαστον* (1080a17–18).[8] That is to say that (a, ii) cannot have a serial order of numerical elements—the very essence of incomparable numbers. In 1080a30–33 Aristotle himself tacitly denies this property to mathematical number when he says that one such number includes another. This is itself a consequence of the statement that in mathematical number all the units are comparable and undifferentiated,[9] and we are therefore entitled to infer that the same thing would be true of numbers such as (a, ii).

Now it is improbable that Aristotle, having mentioned in 1080a18–20 the incomparable monads, went then into a digression concerning the nature of the monads as such, in which the question of the different kinds of numbers was lost sight of; for, though such an interpretation would make sense in itself, Aristotle could hardly have disregarded the fact that 1080a20–21 is still affected by *καὶ τοῦτο κτλ.* in 1080a18.[10] One must then agree with Ross's view that in 1080a20–21 Aristotle does mention incomparable numbers with the units all comparable, but his contention that Aristotle has confused this number with mathematical number must be rejected. The words *οἷον λέγουσιν εἶναι τὸν μαθηματικὸν ἀριθμόν* do not support Ross's interpretation, since this sentence in all probability refers merely to the units' being all comparable, not to incomparable numbers as such.

Why then does Aristotle mention incomparable numbers with the units all comparable, a notion which is self-contradictory as he himself implies? If the text is basically sound, as it seems to be, I submit that he does so for the following reasons. In view of the purely

[8] It is remarkable that G. Reale (*Aristotele, La Metafisica*, traduzione, introduzione e commento II [Napoli 1968] 369 n.7), having seen this last point, nevertheless accepts Ross's interpretation.

[9] *Cf. Metaph.* 1080a22–23.

[10] This is fatal to the interpretation of Robin, *op.cit.* (*supra* n.2) 272–73 n.258, who tacitly denies that in 1080a20–21 Aristotle refers to incomparable numbers with the units all comparable. He reads into 1080a17–23ff the following classification: (1°) Incomparable numbers (1080a17–18), (A) with units all incomparable (1080a18–20); (2°) mathematical number with the units all comparable (1080a20–23); (1° B) incomparable numbers with the units of each number comparable with each other but incomparable with those of other numbers (1080a23ff). (Nor is it possible, I think, to consider lines 20–23 as purely parenthetical.)

theoretical nature of his classification in 1080a15–37 and of his refutation of the separate existence of numbers in 1080b37ff, Aristotle felt he had to mention *all* the possible views of numbers as separate substances and as causes of existing things.[11] So far as we know, no Platonist did believe in incomparable numbers with the units all comparable; but neither did anyone—according to Aristotle himself (1080b8–9)—ever posit incomparable numbers with the units all incomparable. Aristotle's classification in 1080a15–37 is merely for the purpose of a dialectical attack against the diverse Platonistic doctrines of number; it enables him to argue that if numbers actually exist apart from the sensibles, they must belong to one or another of the three categories of incomparable numbers he has set up, all of which he believes to be impossible. Thus in 1080b37ff he tries to prove that separately existing numbers would have to be constituted by incomparable monads such as (a, i) or (a, iii) and that neither can be the case. If the units are all comparable, however, such a number can only be mathematical number, and mathematical number cannot be incomparable;[12] therefore, it cannot have separate existence either.[13]

Now there is some evidence that in rejecting (a, ii) Aristotle wished to indicate what to him was the absurd implication of Speusippus' doctrine and perhaps also to forestall a modified version of Xenocrates' idea-numbers. Speusippus posited the *separate* existence of mathematical number, and Aristotle—who also identifies number with mathematical number—attacks him because of his attempt to 'separate' such a number. Thus in 1083a20–35[14] Aristotle argues that if only mathematical number exists, it cannot exist apart from the sensibles; for, if it did, not only would there have to be a first 'one' (as Speusippus is said to have believed), but there would also have to be a first 'two' and a first 'three', etc. (as, according to Aristotle, Speusippus did not believe); in this case, however, Plato's view of number would be the correct one, since these numbers would have

[11] *Cf. Metaph.* 1080b4–11.

[12] *Cf. Metaph.* 1081a5–12, especially a5–7 (*εἰ μὲν οὖν πᾶσαι cυμβληταὶ καὶ ἀδιάφοροι αἱ μονάδεc, ὁ μαθηματικὸc γίγνεται ἀριθμὸc καὶ εἷc μόνοc*), which is an inference from 1080a22–23 and 30–33.

[13] *Cf. Metaph.* 1083a20–35 and my comments on this passage in the text *infra*.

[14] *Cf.* also *Metaph.* 1081a5–12. That 1083a20–35 refers to Speusippus is shown by comparison of 1080b14–16 with 1028b21–24, 1075b37–1076a3, and 1090b13–20.

to be incomparable numbers. In short, Speusippus' view that numbers with the units all comparable have *separate* existence would amount to the absurd notion of incomparable numbers with the units all comparable; for according to Aristotle, unless such numbers are incomparable, they cannot have separate existence.

Similarly, the absurdity of a number such as (a, ii) may have its use against an attempt to defend a modified version of Xenocrates' view of number. By identifying the ideas with mathematical numbers Xenocrates was most probably trying to offer a compromise between Plato's ideas and Speusippus' mathematical numbers. In fact, however, Xenocrates believed in numbers of the class (a, iii);[15] hence Aristotle's contention that this view destroys mathematical number.[16] Since Xenocrates nevertheless called his ideal numbers mathematical, Aristotle's mention of the possibility of incomparable numbers (something that the Xenocratean idea-numbers would have to be) with the units all comparable forestalls any attempt to defend Xenocrates on the ground that his idea-numbers are really mathematical numbers with the units all comparable.

Thus in 1081a5–7 Aristotle maintains that if all the units are comparable and undifferentiated we get only one kind of number—mathematical—and the ideas cannot be the numbers;[17] conversely, in 1083a17–19 he insists that if the ideas are numbers the units cannot all be comparable.

We must still determine the meaning of 1080a35–37 and its relation to 1080a17–35. The view described in 1080a35–37 cannot be "one which believes in the existence of three *complete* number series of different kinds," as Ross[18] and others believe it is. For it is ostensible from what follows in the rest of chapter 6 of *M* that, though theoretically (a), (b) and (c) are three possible views of number according

[15] On Xenocrates' identification of the ideas with mathematical numbers *cf. Metaph.* 1080b22–23 and 28–30 (with Ross's notes on 1080b22–29), 1028b24–27 (with Ross's notes on 1028b24 and 26–27), 1069a35 (with Ross's note on 1069a34–36), 1076a20–21 (with Ross's note *ad loc.*); Ross, *op.cit.* (*supra* n.1) I lxxiv–lxxv. On Xenocrates' belief in incomparable numbers of the class (a, iii) *cf. Metaph.* 1080b22–23 and 28–30, where *n.b.* *οὔθ' ὁποιαςοῦν μονάδας δυάδα εἶναι.*

[16] *Cf. Metaph.* 1083b1–8 and 1086a5–11, where *n.b.* *τὸν αὐτὸν εἰδητικὸν καὶ μαθηματικὸν ἐποίηςαν ἀριθμὸν τῷ λόγῳ, ἐπεὶ ἔργῳ γε ἀνῄρηται ὁ μαθηματικός, κτλ.*

[17] Aristotle's inference (1081a12ff) that if the ideas are not numbers they cannot exist at all, apart from being unjustified in itself, is vitiated by his misconception about the true nature of number. *Cf.* the references in n.3 *supra* and the corresponding remarks in the text.

[18] *Cf.* Ross, *op.cit.* (*supra* n.1) II 427, and similarly p.426.

to the nature of the respective units, in fact no one has ever held (a),[19] some have held (b),[20] someone (c),[21] others (b) and (c),[22] and still others have identified (b) and (c).[23] And so it would have been pointless and inconsistent to have said in lines 35–37 what Ross thinks Aristotle did say, since, apart from having omitted three different combinations of numbers, Aristotle later states that no one ever posited (a) and never mentions anyone who posited (a), (b) and (c). In short, Ross's interpretation destroys the rationale of Aristotle's classification both in itself and in the light of what follows in the rest of *Metaphysics M*.

There is an alternative interpretation of these lines, however; and that is to take τὸν μὲν . . ., τὸν δ' . . ., τρίτον δέ as introducing three different conceptions of number. But these numbers have already been mentioned in 1080a17–35, so that the ἤ in 1080a35 can hardly introduce the second part of the classification which begins in line 17. This ἤ must be corrective; it introduces a summary but more correct account than that given in 1080a17–35.[24] There is a break in the sentence which begins in line 17, and the anacoluthon leaves the ἤτοι there without its complement. In other words, in 1080a35–37 Aristotle comes back to his original purpose of stating how many *kinds of numbers* can be conceived by those who believe that numbers are separate substances and the causes of existing things. He begins once more with the kind of number described in 1080a17–20, and it is noteworthy that lines 35–37 are still dependent on ἀνάγκη δ' κτλ. in 1080a15–16. But now that the three kinds of monads have been described, there remains to distinguish three possible views of number according to the nature of the component units: (a) incomparable

[19] *Cf. Metaph.* 1080b8–9 and 1081a35–36.

[20] These are Speusippus and the Pythagoreans. *Cf.* 1080b14–21 with Ross's note on 1080b14 and n.14 *supra*.

[21] This is the anonymous Platonist of *Metaph.* 1080b21–22 (cf. Ross's note on line 21), and *n.b.* that according to Aristotle no one believed in incomparable numbers with the units all incomparable, *cf.* n.19 *supra*.

[22] This is the view Aristotle ascribes to Plato. *Cf. Metaph.* 1080b11–14 with 987b14–18.

[23] The view implicitly ascribed to Xenocrates, *cf.* n.15 *supra*.

[24] Robin, *op.cit.* (*supra* n.2), also interpreted lines 35–37 as a résumé of lines 17–35; but, because of his interpretation of lines 20–23 (*cf.* n.10 *supra*), failed to see that the ἤ of line 35 introduces a *corrective* summary of the previous classification. On corrective ἤ at the beginning of clauses (Kühner-Gerth, *Griechische Grammatik* II p.297 #3) even when no question precedes, *cf.* Arist. *Top.* 159a11, *Eth.Nic.* 1100b7; H. Bonitz, *Index Aristotelicus* 313a17–26, esp. 25–26.

numbers with units all incomparable, (b) mathematical number with units all comparable, (c) incomparable numbers with the units of each number comparable with each other but incomparable with those of other numbers. Given that 1080a20–23 and 1080a30–33 contain an implicit refutation of the possibility of numbers such as (a, ii),[25] it suffices for Aristotle here to mention as the second kind of number mathematical number, according to him the only kind of number that is possible if all the units are comparable and undifferentiated. After chapter 6 Aristotle rejects (a) and (c) and in the case of (b) tries to show that mathematical number, precisely because its units are comparable and undifferentiated, cannot exist apart from the sensibles.

To summarize the results of the preceding discussion: Aristotle begins in 1080a17 as if his classification of numbers as separate substances and as causes of existing things were going to be: on the one hand either (a, i) or (a, ii) or (a, iii), and, on the other hand, (b). But, because in maintaining (a, ii) he mentions mathematical number as an example of number with the units all comparable and undifferentiated and because after mentioning (a, iii) he goes into a rather lengthy digression to explain the difference between mathematical number and incomparable numbers such as (a, iii), he probably felt that it would be anticlimactic to mention (b) as the second and final part of his classification. Hence the break in the construction which begins with ἤτοι κτλ. in 1080a17 and the corrective and summary classification of 1080a35–37. The most serious difficulty in 1080a15–37 is one of contorted syntax, not of conceptual confusion as Ross thinks.

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[25] *Cf.*, with the corresponding remarks in the text, n.12 *supra*.