

# Eratosthenes in the *Excerpta Neapolitana*

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THE AIM of the present note is to discuss the procedure for finding semitones and quarter-tones that is ascribed to Eratosthenes in the so-called *Excerpta Neapolitana*.<sup>1</sup> We shall show, by carefully assessing the manuscript tradition of the *Excerpta*, that such a piece of information cannot be directly used as evidence of any sort to reconstruct Eratosthenes' elaborations in harmonic theory: thus, the procedure for finding semitones and quarter-tones expounded in the *Excerpta Neapolitana* may well originate with Eratosthenes, but what we read in the *Excerpta* does not corroborate or refute this thesis.

We shall first (§1) explain some basics of the Greek divisions of the monochord, including the evidence about Eratosthenes' divisions. We shall then (§2) present the harmonic material contained in the *Excerpta Neapolitana* and (§3) discuss their layout in what is nowadays their main witness, the manuscript Città del Vaticano, Biblioteca Apostolica Vaticana, *Vat.gr.* 2338; this discussion will lead us to a final assessment (§4) of the *Excerpta*'s pertinence to Eratosthenes' canonic divisions. An Appendix will present a collation of the *Excerpta Neapolitana* in *Vat.gr.* 2338 with their standard critical text.

## 1. *Eratosthenes' divisions of the monochord*

A division of the monochord, or canonic division, is a rational method for locating notes on a harmonic interval of a given

<sup>1</sup> These abbreviations are used: *AEH* = R. Da Rios, *Aristoxeni Elementa Harmonica* (Rome 1954); *MSG* = C. von Jan, *Musici Scriptores Graeci* (Leipzig 1895). With the exception of the *Neapolitanus* (a reproduction of f. 43<sup>r</sup> is *fig.* 1), online reproductions of all manuscripts here mentioned can be found through <https://pinakes.irht.cnrs.fr/>. The perceptive remarks of the referee greatly contributed to improving our argument. Sara Panteri thanks the participants in the conference Harmonic Theory in Ancient Greece (Berlin, April 2018), where a portion of the material discussed in this paper was first presented.

span. Several procedures were put forward in Greek antiquity to locate the notes; individual authors and harmonic schools differed as to the rational tools to be used to carry out the division, and of course as to the exact location of the notes. If we allow ourselves, as one of those schools did, to use numerical ratios to represent musical intervals—by no means an innocent move, as we shall see—and focus on an interval of one octave, this is compounded of a fifth and a fourth:  $2:1 = (3:2)(4:3)$ , while the compounding notes differ by an epogdoic tone:  $(3:2):(4:3) = 9:8$ . Thus, an octave is compounded of two intervals of a fourth located a tone apart:  $2:1 = (4:3)(9:8)(4:3)$ . These three conjoined intervals are bounded by four notes: these are the “standing” (or “fixed”) notes in the octave. Each of the intervals of a fourth within the octave was called a “tetrachord”; they are said to be “disjoined” by the tone intervening between them. One of the essential tasks of Greek harmonic theorists was to locate two more notes within each tetrachord;<sup>2</sup> these are the “movable” notes, whose different positions produce the basic genera (enharmonic, chromatic, and diatonic) and their shades.<sup>3</sup> Traditionally, both “standing” and “movable” notes within the two disjoined tetrachords making an octave were made to correspond by placing them one fifth apart;<sup>4</sup> thus, only the movable

<sup>2</sup> This is one of the main points of Aristoxenus’ agenda: see *El.harm.* 4.25–28 and 35.1–4 Meibom.

<sup>3</sup> See for instance Aristoxenus’ discussion at *El.harm.* 22.24–27.14 and 50.15–52.33; Aristoxenus refers to a τόπος “range” within the tetrachord where each movable note can be located. As for the double reference just cited, recall that Aristoxenus’ *El.harm.* 1 and 2 sometimes cover the same material; for this reason, we shall henceforth refrain from referring to parallel passages in the two books. For the problems raised by the structure of the treatise see A. Barker, *The Science of Harmonics in Classical Greece* (Cambridge 2007) ch. 5.

<sup>4</sup> This is Aristoxenus’ fundamental principle, formulated at *El.harm.* 54.2–7; see also its immediate consequence, at 58.6–59.5, on contiguous tetrachords being conjoined or disjoined—that is, either having a note at one of the extremities in common or being separated by a tone.

notes within either of the tetrachords had to be located. The outcome of this procedure is a canonic division over the span of one octave; the structure can be enlarged by suitably joining other tetrachords to this central octave. The so-called “systems” were thus produced: these are sequences of tetrachords possibly disjoined by a tone.<sup>5</sup>

Several sources allow us to reconstruct the canonic divisions proposed by individuals or by harmonic schools. One of the most valuable, albeit very terse, such sources is chapters 2.14–15 of Ptolemy’s *Harmonica*.<sup>6</sup> these comprise a long series of tables setting out several canonic divisions over the span of the octave, sometimes with indication of their author; the tables are accompanied by a short and merely descriptive text.<sup>7</sup> Ptolemy needed a uniform representation by numbers in order to compare the canonic divisions he surveys with one another and with his own. To do this, he did not represent the division in ratios,

<sup>5</sup> Such additional tetrachords are in fact usually “conjoined” (see n.4) to the original ones; in this case the interval between corresponding notes is a fourth. A further tone is also usually attached to the lowest tetrachord. On Greek divisions of the monochord see A. Barker, “Three Approaches to Canonic Division,” *Apeiron* 24.4 (1991) 49–83; Barker, *The Science* 12–18, with very clear diagrams; and D. E. Creese, *The Monochord in Ancient Greek Harmonic Science* (Cambridge 2010).

<sup>6</sup> Edition in I. Düring, *Die Harmonielehre des Klaudios Ptolemaios* (Göteborg 1930) 70–80, English translations with commentaries in A. Barker, *Greek Musical Writings II* (Cambridge 1989) 346–355, and J. Solomon, *Ptolemy, Harmonics* (Leiden 2000) 97–123.

<sup>7</sup> The text of a part of chapter 2.14 is missing in the entire manuscript tradition; the late-fourteenth-century Byzantine scholar Isaak Argyros restored it in his recension, autograph in the MS. *Vat.gr.* 176, of Ptolemy’s treatise. Argyros took his model in the surviving portion of the chapter, his restoration just amounting to a back-translation into natural language of the numbers set out in the tables (from the philological point of view, there is no difference in the likelihood of corruptions having occurred in numerals set out in tables or within a text). Argyros’ recension is in its turn grounded on Gregoras’ recension of the *Harmonica*. See Düring, *Die Harmonielehre* LIX–LXVI and LXXVIII–LXXXIX, on all this.

but in string lengths relative to a common measure: as Ptolemy himself explains at the end of *Harm.* 2.13, he partitions his monochord into 120 units, assigning values 60 and 120 to the highest and to the lowest note bounding an octave, respectively. As a consequence, the other two standing notes in any canonic division are assigned the numbers 80 and 90.<sup>8</sup> In this framework, the numbers assigned to the four movable notes distinguish one canonic division from another. For instance, the enharmonic genus according to Aristoxenus is mapped onto the sequence of numbers 60, 76, 78, 80, 90, 114, 117, 120. The same numbers identify the same genus according to Eratosthenes. Aristoxenus' tonic chromatic genus is also identical to Eratosthenes' chromatic, the numerical sequence being in this case 60, 72, 76, 80, 90, 108, 114, 120. The sequence associated with Eratosthenes' diatonic—namely, 60, 67.30', 75.56', 80, 90, 101.15', 113.54', 120—is instead identical to Ptolemy's ditonic diatonic.<sup>9</sup>

Since these sets of values entail an awkward conversion of Aristoxenus' additive system of tonal distances into the numerical language typical of the Pythagorean approach in terms of ratios,<sup>10</sup> modern scholarship has debated about the pertinence of such a conversion and about its author. On the basis of what we read in Ptolemy, the name of Eratosthenes most naturally

<sup>8</sup> For  $80:60 = 4:3$  (a fourth), and  $120:80 = 90:60 = 3:2$  (a fifth).

<sup>9</sup> This canonic division is the one adopted in *Sect.can.* 19–20 and implied in Philolaus fr.6a and in Pl. *Ti.* 35B–36B. We use Ptolemy's sexagesimal notation; two of Düring's conversions into fractions are incorrect. Note also that the partitions of the tetrachords set out in the last row of Düring's tables are his own addition; all translations reproduce these partitions. This remark will be useful at the end of §3.

<sup>10</sup> In the text accompanying the first table of *Harm.* 2.14 (71.2–3 Düring) Ptolemy is careful in setting out the main partition of Aristoxenus' tetrachord in terms of the intervals 24, 3, 3, and qualifies it as being ἐν διαστάσει “in distance[s]”, whereas Eratosthenes' is ἐν λόγοις “in ratios.” Here διάστασις must mean “musical interval,” as in some other occurrences in the *Harmonica*: see the index entry in Düring, *Harmonielehre* 129 s.v., and the several renderings in e.g. Barker, *Greek Musical Writings* II.

comes to the fore. This learned debate is relevant to our present purposes because it involves the main piece of a threefold *direct* evidence about Eratosthenes' canonic divisions.<sup>11</sup> The other two pieces of evidence are a passing remark in Nicomachus' *Encheiridion*,<sup>12</sup> which will not detain us any longer, and a section of the so-called *Excerpta Neapolitana*.

## 2. *The Excerpta Neapolitana*

The *Excerpta Neapolitana* are a structured set of pieces of Greek musical lore apparently excerpted from a number of sources, some of which can be identified. They are contained, under the title Πτολεμαίου μουσικά, in ff. 41<sup>v</sup>–45<sup>r</sup> (*fig. 1: f. 43<sup>r</sup>*) of the first codicological unit (end fifteenth century) of the MS. Napoli, Biblioteca Nazionale Centrale Vittorio Emanuele III, III.C.2, once owned, and maybe partly commissioned, by the renowned Renaissance scholar Giorgio Valla.<sup>13</sup> The excerpts, which C. von Jan could read in this very manuscript (whence the name), were first studied by him in a 1894 article on the harmony of the spheres, and then published in his classic 1895 edition of the *Musici Scriptores Graeci*.<sup>14</sup>

<sup>11</sup> Most sustained discussions of the evidence in Creese, *Monochord* 178–209; S. Hagel, *Ancient Greek Music. A New Technical History* (Cambridge 2010) 182–187; see also A. Hicks, “Reinterpreting an Arithmetical Error in Boethius's *De institutione musica*,” *Music Theory and Analysis* 3 (2016) 1–26, at 19–21. Other passages in e.g. Theon of Smyrna and Porphyry refer to Eratosthenes' harmonic doctrines, but do not provide direct information on his canonic divisions.

<sup>12</sup> At *MSG* 260.12–17, English transl. Barker, *Greek Musical Writings* II 266.

<sup>13</sup> Valla used this manuscript for his Latin translation of Cleonides' *Introductio harmonica*, printed in 1497 in Venice. The two codicological units of *Neapol.* III.C.2 comprise ff. 1–54 and 55–100, respectively. The second of them was penned by John Rhosos between 1468/1472 and 1498, most likely in the 1490s (watermarks). See the catalogue entry in M. R. Formentin, *Catalogus codicum graecorum Bibliothecae Nationalis Neapolitanae* III (Rome 2015) 87–89.

<sup>14</sup> C. von Jan, “Die Harmonie der Sphären,” *Philologus* 52 (1894) 13–37, and *MSG* 411–420.

In the Naples manuscript, the *Excerpta* are divided into sections by means of rubricated initial letters; the edition in *MSG* introduces a finer partition (more on this below). Some sections are set out by means of a layout nicely organized in two columns within a window in the text page. As for the sources, von Jan identified a long verbatim quote from Cleonides' *Introductio harmonica* (sections 17–18).<sup>15</sup> The *Excerpta* also feature two long sections elsewhere, (unwarrantedly) identified as extracts from Nicomachus (section 23 and a section to which von Jan assigns number 28 only in his introduction to the *Excerpta* at *MSG* 409–410), and Aristoxenian material (sections 4–15) other than the excerpts from Cleonides.<sup>16</sup> Many sections of the *Excerpta* simply contain lists of definitions of terms relevant to harmonic theory (the subject-matter of sections 4–8, 17–19, 23, 27–28) or to rhythmic theory (9–15, 20–22), and even pertaining to Aristotelian natural philosophy (16 and 26). In sections 1–3 and 24–25 one finds material on the harmony of the spheres, the former segment possibly drawing from Pythagorean doctrines,<sup>17</sup> the latter virtually identical (apart from copying mistakes) to two sections of Ptolemy's *Canobic Inscription*.<sup>18</sup> Exactly these sections

<sup>15</sup> Cleonides' text is *MSG* 192.12–193.2; this is the final segment of ch. 7 of Cleonides' treatise.

<sup>16</sup> Sections 4–8, 17, 18 are accepted as *testimonia* in *AEH* and numbered 21, 22, 35, 37, 48, 57, 96; see also the analysis in L. Zanoncelli, *La manualistica musicale greca* (Milan 1990) 467–492. Von Jan did not print some of the sections extracted from writings (Cleonides and the pseudo-Nicomachus) that he presents elsewhere in *MSG*; curiously, he did print section 17 but not 18, even though both are verbatim excerpts from Cleonides.

<sup>17</sup> For a comparison of sections 1–3 with the identical piece of Neo-Pythagorean arithmological lore in Anatolius as attested in [Iambl.] *Theol.ar.* 75.8–76.4 De Falco, see A. J. H. Vincent, *Notice sur divers manuscrits grecs relatifs à la musique* (Paris 1847) 251.

<sup>18</sup> Edition of the *Canobic Inscription* in A. Jones, "Ptolemy's *Canobic Inscription* and Heliodoros' Observation Reports," *SCLAMVS* 6 (2005) 53–97; most recent and comprehensive study in N. M. Swerdlow, "Ptolemy's *Harmonics* and the 'Tones of the Universe' in the *Canobic Inscription*," in Ch. Burnett et

were first studied by von Jan in his 1894 article; a comparison with Ptolemy's statements on the same subject in *Harmonica* 3.16 was in fact in order because of the title that the *Excerpta* carry in the Naples manuscript.

Eratosthenes comes on stage in section 19. The immediately preceding two-section extract from Cleonides expounds the Aristoxenian<sup>19</sup> division of the tone into twelve equal parts, and accordingly the division of the semitone into six parts, and of the diesis (intended either as a fourth or as a third part of a tone) into three or four parts. Then we read a single sentence: ἔστι δὲ ἡ εὕρησις τῶν τόνων καὶ τῶν ἡμιτονίων καὶ τῶν διέσεων κατὰ τὸν Ἐρατοσθένην, “and then there is the finding of the tones, the semitones, and the dieseis according to Eratosthenes,” followed by a series of diagrams + captions showing exactly how to find semitones and the enharmonic dieseis of a quarter tone: this is done by recursively rescaling (namely, multiplying both terms of) the ratio 9:8 by 2 (yielding 18:16) and by 4 (yielding 36:32), and by accordingly inserting the middle terms 17 (between 18 and 16) and 33 and 35 (between 36:32 with middle term 34 = 17×2), respectively. That is all; *exit* Eratosthenes.

Modern scholarship has quite naturally compared these numbers with the triad 120, 117, 114, representing, as we have seen in the previous section, the three lowest notes in Aristoxenus'/Eratosthenes' enharmonic division of the tetrachord. For these notes bound the two intervals making a so-called enharmonic *pyknon*:<sup>20</sup> the numbers on which they are mapped stand in the ratios 120:117 and 117:114, which are equal to 40:39 and to 39:38, respectively; the enharmonic *pyknon* itself is mapped on the ratio 120:114 = 40:38 = 20:19, and can be obtained from

al. (eds.), *Studies in the History of the Exact Sciences in Honour of David Pingree* (Leiden 2004) 137–180, in particular at 165–176.

<sup>19</sup> This we gather from Porph. *In Harm.* 125.24–126.2 Düring; cf. Aristox. *El.harm.* 21.20–31 and 25.11–26.5; in the latter passage, intervals of a twelfth of a tone are apparently assumed as basic counting-units.

<sup>20</sup> The definition of a *pyknon* is at Aristox. *El.harm.* 24.11–14.

Eratosthenes' chromatic *pyknon* 10:9 by rescaling it by 2 and then inserting the middle term 19 in the resulting ratio 20:18.

The rationale behind both procedures is that rescaling a superparticular<sup>21</sup> ratio in lowest terms by 2 gives room to insert exactly one integer middle term, thereby producing two superparticular ratios in lowest terms that “divide” the original interval;<sup>22</sup> iterating the procedure allows inserting further integer middle terms and generating further superparticular ratios in lowest terms, each dividing one of the intervals generated in the previous step. In the language of the multiplicative approach to canonic division, this amounts to dividing an interval by replacing each time a geometric mean (as should be done in order to divide an interval into two *equal* intervals) by an arithmetic mean.<sup>23</sup> Conversely, since Aristoxenus' enharmonic tetrachord is additively divided into ditone + quarter-tone + quarter-tone—which correspond,<sup>24</sup> adopting the Aristoxenan division of the

<sup>21</sup> Of course, superparticular ratios are not the only class of ratios to which this procedure—which P. Tannery, “Sur les intervalles de la musique grecque,” *REG* 15 (1902) 336–352, repr. *Mémoires Scientifiques* II (Toulouse-Paris 1912) 97–115, at 105, calls “division harmonique,” and R. P. Winnington-Ingram, “Aristoxenus and the Intervals of Greek Music,” *CQ* 26 (1932) 195–208, at 198, dismisses as “naïve”—can be applied: see e.g. Boeth. *Mus.* 3.8 and 4.6.

<sup>22</sup> The meaning of “divide” in this context is that the original ratio is compounded of the two ratios arising from the insertion of the middle term. For instance, 18:16 is compounded of 18:17 and 17:16 (cf. Euc. *Elem.* 6.def.5); note that the two compounding ratios are *nearly* equal. To divide, in the sense just described, a superparticular ratio into two equal intervals is impossible, as Archytas first proved: see further below.

<sup>23</sup> Using the arithmetic mean to approximate the geometric mean of two numbers is the gist of the Heronian iterative method to compute an approximate square root of an assigned number. See F. Acerbi and B. Vitrac, *Héron d'Alexandrie, Metrica* (Pisa/Rome 2014) 121.

<sup>24</sup> See e.g. *El.harm.* 46.30–32 and 50.22–25. On Aristoxenus' intervals, described at *El.harm.* 22.24–27.14 and 50.15–52.33, see Tannery, *Mémoires* 97–115 (Tannery sets out a table analogous to those in *Harm.* 2.14, the



tone into twelve equal parts attested in Cleonides, to a numerical sequence of interval sizes 24, 3, 3—once this division is mapped onto Ptolemy’s 120-unit gauge within the lowest tetrachord (90,120), it yields exactly the numbers 114, 117, 120 set out in Ptolemy’s table as the positions of the lowest notes bounding the corresponding intervals. But then, since these same tables allow for a uniform reading only if viewed within the multiplicative approach endorsed by Ptolemy, this entails a distortion of the Aristoxenian system, which amounts to replacing each time an arithmetic mean by a geometric mean.<sup>25</sup> A mathematical inconsistency arises in this way, but one must admit that Ptolemy—or Eratosthenes before him—did not have any alternative to introducing a bug somewhere in the mapping: providing a unified view of empirical and mathematical harmonics on the issue of interval lengths is simply impossible.<sup>26</sup>

Leaving aside this inconsistency, comparison of the series of diagrams + captions in the *Excerpta Neapolitana* and of the evidence from Ptolemy’s table explained in the previous paragraph has quite naturally led scholars to assert that, despite some differences (after all, the numbers set out are not exactly the same), “the two intervals of [Eratosthenes’] enharmonic *pyknon* are generated according to a procedure for ‘halving’ ratios attributed to [him] in the *Excerpta Neapolitana*.”<sup>27</sup>

This is the statement we want to question. For the *Excerpta Neapolitana* are a reasonably tightly-structured set of excerpts: a typical Late-Antiquity compilation, possibly the outcome, in its present form, of some accretions from a core that we have every reason to suppose not decidedly smaller than the present text.

numbers being calculated according to the additive model); Winnington-Ingram, *CQ* 26 (1932) 195–208; Hagel, *Ancient Greek Music* 151–158.

<sup>25</sup> This is clear already from the (tendentious) argument in Ptol. *Harm.* 1.9.

<sup>26</sup> The difference arising from replacing the arithmetic mean of two given numbers with their geometric mean (or vice versa) is smaller the further one proceeds in the iterative algorithm described in this paragraph, and eventually non-perceptible: Tannery, *Mémoires* 104–106; Creese, *Monochord* 193.

<sup>27</sup> Creese, *Monochord* 190–193; quotation from 191.

Now, one of these accretions is in fact the entire “section” where Eratosthenes is mentioned. Let us see why, and how this has probably happened.

### 3. *The Excerpta Neapolitana in Vat.gr. 2338*

The manuscript *Vat.gr.* 2338 is a “receuil factice” made of four codicological units: ff. 1–22, 23–38, 39, 40–44. The first codicological unit (late thirteenth century), and only this, contains treatises and extracts of Greek harmonic theory: Cleonides, *Introductio harmonica*; Euclid, *Sectio canonis*; Gaudentius, *Introductio harmonica*; Theon of Smyrna, *excerpta musica* (*Exp.* 46.20–57.6 Hiller); [Pappus], *inmo* Cleonides, *Introductio harmonica*; Aristoxenus, *Elementa harmonica* 1–3; and finally, on ff. 21<sup>v</sup>–22<sup>v</sup>, the *Excerpta Neapolitana*.<sup>28</sup>

*Vat.gr.* 2338 was unknown to C. von Jan: the manuscript was brought into the Vatican Library by Cardinal Giovanni Mercati, from the church of Sant’ Andrea della Valle in Rome. This manuscript plays a major role in the transmission of the Greek musical writings it contains, as was first recognized by R. Da Rios in her 1954 edition of Aristoxenus’ *Elementa harmonica*. As Da Rios first showed, there is not the slightest doubt that the first codicological unit of *Neapol.* III.C.2 is a direct, and in fact slavish, copy of *Vat.gr.* 2338.<sup>29</sup> Had von Jan known of *Vat.gr.* 2338, he would probably have called the *Excerpta Neapolitana Excerpta Vaticana*. Since the other manuscript witness used by von Jan, Paris, Bibliothèque nationale de France, *gr.* 3027, is also a

<sup>28</sup> On the manuscript see *AEH* XX–XXV; T. J. Mathiesen, *Ancient Greek Music Theory. A Catalogue Raisonné of Manuscripts* (Munich 1988) 608–611 no. 234; the companion paper F. Acerbi and A. Gioffreda, “*Harmonica Membra Disiecta*,” *GRBS* 59 (2019) 646–662.

<sup>29</sup> *AEH* LXXV and LXXIX–LXXXI. As for Cleonides, see J. Solomon, *Cleonides: Εἰσαγωγή ἁρμονικὴ* (diss. Univ. North Carolina 1980) 60–99, and “Vaticanus gr. 2338 and the *Εἰσαγωγή ἁρμονικὴ*,” *Philologus* 127 (1983) 247–253. As for Euclid, see A. Barbera, *The Euclidean Division of the Canon. Greek and Latin Sources* (Lincoln 1991) 67–68.

copy of *Vat.gr.* 2338,<sup>30</sup> we may focus on the latter in our discussion of section 19 of the *Excerpta Neapolitana*.

The crucial fact to be remarked is that only the sentence “and then there is the finding of the tones, the semitones, and the dieseis according to Eratosthenes” is included in the main text, whereas the series of diagrams + captions is located, *penned by the main hand*,<sup>31</sup> in the outer margin. Since *Neapol.* III.C.2 preserves the layout of its model,<sup>32</sup> this crucial piece of information was also provided by von Jan, but has apparently been neglected in all subsequent scholarship.<sup>33</sup>

If instead we do not neglect this piece of information, a global picture of the formation of the *Excerpta Neapolitana* begins to take shape, as follows. A consistent core of excerpts of harmonic and rhythmic theory was collected in Late Antiquity, possibly laid out in two columns as was customary in manuscripts of that period, in a codex that already contained treatises on the same subject. The core of the collection we take to be sections 4–15, 17–18, 20–23, 28.<sup>34</sup> Other sets of extracts, unrelated to one another, were added in the margins, intended to supplement this material with independent pieces of information. These are the

<sup>30</sup> By the intermediation of Cambridge, University Library, gr. 1464: Mathiesen, *Ancient Greek Music Theory* 252, and Barbera, *The Euclidean Division* 67–68.

<sup>31</sup> The scholium summarizing part of section 17, penned in red ink in *Neapol.* III.C.2 and edited in *MSG* 416.6–9 app., is copied from *Vat.gr.* 2338, where it was apposed at the end of the fourteenth century by Philotheos of Selymbria (*Prosopographisches Lexikon der Palaiologenzeit* 29896).

<sup>32</sup> With some omissions: Valla supplied the text at *MSG* 417.6; this shows that he checked the copy against the original.

<sup>33</sup> In describing *Vat.gr.* 2338, Mathiesen, *Greek Music Theory* 610, misleadingly asserts that “a lengthy lacuna follows Ἐρατοσθένην, omitting the diagrams and text (Jan 416.14–417.11); the missing material appears in the margin.”

<sup>34</sup> The partition into sections in *Vat.gr.* 2338 as indicated by rubricated initials is 1, 2, 3, 4, 5, 6, 7, 8, 9–15, 16 divided into 4 sections, 17–19, 20–22, 23, 24–26, 27, 28.

extracts on the harmony of the spheres (sections 1–3 and 24–25, all in a tabular layout, with 3 possibly being a further scholium to 1–2), on tuning musical instruments (27, again in a tabular layout and possibly completing 23), on natural philosophy (16 and 26, the former almost certainly a scholium to the last line of section 24);<sup>35</sup> and again an incomplete jotting, possibly related to section 22,<sup>36</sup> and the specification about the existence of Eratosthenes’ finding of the tones, the semitones, and the diesis. This is the first sentence of von Jan’s section 19, which we quoted above (ἔστι δὲ ἡ εὐρησις τῶν τόνων καὶ τῶν ἡμιτονίων καὶ τῶν διέσεων κατὰ τὸν Ἐρατοσθένην) and which we find included in the main text—and after a space clearly separating it from what precedes—in *Vat.gr.* 2338.<sup>37</sup> What precedes is the excerpt from Cleonides. In it, as we have seen, he expounds the Aristoxenian division of the tone into twelve equal parts, of the semitone into six parts, and of the diesis into three or four parts. On this basis, Cleonides also sets out the numerical sequences of interval sizes associated with several divisions of the octave: enharmonic, 24, 3, 3; soft chromatic, 22, 4, 4; hemiolic chromatic, 21, 4<sup>1/2</sup>, 4<sup>1/2</sup>; tonic chromatic, 18, 6, 6; soft diatonic, 15, 9, 6; tense diatonic, 12, 12, 6. Of course, these are Aristoxenus’ divisions: but in chapter 2.14 of Ptolemy’s *Harmonica*, two of them turn out to be mapped onto numerical sequences identical to divisions ascribed to Eratosthenes—and this leads us again to the specification opening section 19 of the *Excerpta*: it is a reader’s gloss to the *Excerpta* jotted down in the margin of a manuscript that might well have contained Ptolemy’s *Harmonica*, too.<sup>38</sup>

<sup>35</sup> So, in our opinion rightly, Zanoncelli, *Manualistica* 489.

<sup>36</sup> It is τρία γένει τῶν ποδῶν· ἰαμβικὸν, not transcribed in *MSG* and to be read in *Vat.gr.* 2338, f. 22<sup>r</sup> marg. ext., and *Neapol.* III.C.2, f. 43<sup>r</sup> marg. ext. with the correction γένη.

<sup>37</sup> We are grateful to the referee for suggesting to us the scenario that follows.

<sup>38</sup> Thus, the verb “to be” opening the sentence must be copulative, and not existential as in our translation at 669 above: read “but this is the finding of

This specification was further, independently, and not wholly pertinently<sup>39</sup> buttressed by a standard diagrammatic scholium accompanied by short captions: the whole makes up von Jan's section 19. We may even indicate a most plausible source for this scholium: it is Aristid. *Quint.* 3.1,<sup>40</sup> where exactly the same procedure is expounded to find the semitone and the diesis, ascribed generically to people endorsing the mathematical approach to harmonics.<sup>41</sup>

The entire set of excerpts was copied one or several times before surfacing in *Vat.gr.* 2338. The copying process entailed some rearrangements of the core material—among which misplacing sections 20–22 from before 9–15 to their present location<sup>42</sup> and possibly mixing the extracts related to rhythemics with those pertaining to harmonics—and the inclusion of some of the marginal additions in the main text. The extracts on the harmony of the spheres were eventually (that is, when the final layout on a full page was adopted) inserted within “windows” inside the text, their original layout on two columns being retained;

(etc.),” that is, “but this is the same finding as that of (etc.).” As we remarked above (n.9), the original tables do not carry the piece of information on the partitions of the tetrachords set out in the last row of the tables edited by Düring; but such a piece of information is contained in the extant text, and was certainly contained in the missing portion of it (n.7 above). Thus, the sequences of interval sizes listed by Cleonides also featured in *Harm.* 2.14.

<sup>39</sup> It is not explained how to “find” the tone, whereas instructions are provided on how to find the ditone (by compounding 9:8 with itself). Maybe the diagram related to the ditone was even a further addition to the previous diagrams. See also the beginning of §4.

<sup>40</sup> At 95.8–96.17 Westphal; English transl. Barker, *Greek Musical Writings* II 495–496.

<sup>41</sup> Many more sources can be indicated if we restrict ourselves to the semitone: see *Anon. in Platonis Theaetetus* (P.Berl.inv. 9782) cols. 34.46–35.12; *Ptol. Harm.* 1.10; *Plut. De an. procr.* 1021C–E; *Theon of Smyrna Exp.* 69.12–70.1 Hiller; *Procl. In Ti.* II 179.10–180.26 Diehl; *Boeth. Mus.* 3.1, 270.4–18 Friedlein.

<sup>42</sup> So again, in our opinion rightly, Zanoncelli, *Manualistica* 489.

Sections 24–25 were instead adapted to the full page setting, while keeping some longer blank spaces between the items. Only the incomplete jotting related to section 22 and the diagrammatic scholium + captions making part of section 19 were kept in the margin because every copyist—including the one of *Napol.* III.C.2—was able to assign them their correct textual function: paratextual items. The copyists were able to do this because two features characterise these items as paratexts: the first is obviously incomplete; the second has diagrams as its essential component, and diagrams of a kind that amounted to a standard exegetical tool.<sup>43</sup>

#### 4. *Assessment*

Disentangling the two components of section 19 of the *Excerpta Neapolitana* has the obvious consequence that we stated at the beginning: the information provided in that section cannot be directly connected to Eratosthenes' divisions of the monochord. The content of the diagrammatic scholium confirms this: it presents a simple technical trick that was part of the basic knowledge underlying arithmetical harmonic theory at least since Archytas' celebrated theorem on the impossibility of inserting a mean proportional between two terms in a superparticular ratio;<sup>44</sup> this theorem is further attested in prop. 3 of the *Sectio canonis*, the key result about the conservation of the multiplicity of mean proportionals under the operation of rescaling a ratio being proved in *Eucl. Elem.* 8.8. Most importantly, the diagrammatic scholium does not work out the procedure starting from ratio 10:9, as Eratosthenes' tetrachord in Ptolemy's table presupposes, but from the standard epogdoic tone: thus, the diagrammatic scholium just elaborates on the sentence opening

<sup>43</sup> Such diagrams were used in different context: harmonic theory, numerical proportion theory, logistic, Aristotelian syllogistic—namely, every time the three possible relations between three terms had to be represented.

<sup>44</sup> This was already recognized in Tannery, *Intervalles* 105: the “division harmonique” is a “procédé déjà employé par Archytas.” Archytas' theorem is at *Boeth. Mus.* 3.2, 285.9–286.19 Friedlein.

section 19 of the *Excerpta*; any connection with the Aristoxenian or Ptolemaic context is lost.

The fact that, as we have seen, the procedure in the *Excerpta Neapolitana* can be found in Aristides Quintilianus further corroborates our point. Of course, one might surmise that behind the latter's anonymous procedure for finding semitones and quarter-tones there is Eratosthenes, and that, contrary to Aristides Quintilianus himself, the scholiast who added the diagrams to the set of harmonic excerpts in an ancestor of *Vat.gr.* 2338 *knew* that. Granted, but this is only a hypothesis, and as such confirms our contention that this procedure cannot be ascribed to Eratosthenes on the basis of the evidence constituted by section 19 of the *Excerpta Neapolitana*.

#### APPENDIX:

##### A Collation of the *Excerpta Neapolitana* in *Vat.gr.* 2338

We record only variants that are not attested in *Neapol.* III.C.2 (and only in this manuscript) according to von Jan's main text and apparatus; variant readings in the spelling of numerals (numeral letters vs. full word) are disregarded. Corrections *in rasura* are noted "ras.," because we are only interested in the original text of the *Excerpta* in *Vat.gr.* 2338 (siglum A); the corrected text normally coincides with that in the Naples manuscript (N). Unless otherwise stated, long omissions in this manuscript that von Jan did not mark as his own integrations (smaller font in his edition) are mistakes by *saut du même au même* of the copyist: therefore, *Vat.gr.* 2338 does have the sequence. We refer to page.line(s) of *MSG*, even for the two texts (Cleonides and the excerpts from Nicomachus) that von Jan simply mentions by indicating the page-range of his own edition elsewhere in *MSG*.

411.3 οἱ ὄροι 3 οὕτως 4 ἔχ{ras.} ὄγδοον τὸν θ ἀριθμόν, ὑπερέχει  
6 8 ἐπίτριτον 9 ἡμιόλιον 10 διπλασι/ ἐπίτριτος τοῦ η, ὑπεροχὴ γ  
12 η 13 διπλά τοῦ η, ἡμι τοῦ κδ

412.1–8 omnia planetarum signa deficient 1 ras. (μὲν)  
4 {ras.} ὄγδω {ras.} οὗ 8 τοῦ θ, ἀπλανω ἐπογδῶ λόγω  
10 ὑπε{ras.} 14 η ὑπερέχει {ras.} η {ras.} θ μονάδι 20 τελείου

413.1 πρεπόντων e corr. 8 τοὺς δὲ ἀκολούθους 15 ὁ habet A  
16–17 ἡ ἄλογία δὲ ἐν χρόνοις κειμένη τὸν εἰρημένω

- 414.1 ἐνρυθμοὶ 3 ἀλλήλους 5 ἄρυθμοὶ 7 παντὶ καὶ παντὸς ἄγνωστον ἔχοντες πρὸς ἀλλήλους 9 ὁ μείζων 12 ῥυθμοὶ δυνάμενοι 13 ἐπιδείξασθαι, τρεῖς ἴσως, διπλασίων, ἡμιόλιον 15 πεονικόν 16 τετρασίμου 17 ἑξκαιδεκασίμου 19 δισίμῳ μεγέθει 20 {ras. 3 litt.} τρισίμου 21 ὀκτωκαιδεκασίμου 22 ἑξαπλάσιος 23 πεονικὸν ἄρχεται μὲν ἀπὸ {ras. 3 litt.} πεντασίμου
- 415.1 πεντεκαιεικοσασίμου 1–2 om. ὥστε γίνεσθαι τὸν μέγιστον πόδα 6 τρίσιμος 7 τὸ διπλάσιον 10–11 μεγέθη 11 γένη 13–19 legi nequit
- 416.3 τόνος εἰς 11 δωδεκατημόρια
- 192.19 μελωδηθήσεται 21 μαλακῶ 23–193.1 om. ιη — θ καὶ
- 416.15 habent AN
- 417.6 μεταξὺ ἀριθμοὶ 7–8 {ras.} πολλαπλασιαζόμεν {ras.} 18 ἐπὶ τούτοις
- 418.4 ἴσως
- 266.3–4 παρέδωκε 10 Ἄντησαν 11 εὐρόντα 14 προ {ras.} ευρετήν 15 οὕτως 16 Ἄχ {ras.} οὖς
- 418.16 μη
- 419.1 δὲ ἐζευγμένων 3 παράμεσες 5 μέσαν 8 μεσοτήτων μὲν 9 γεωμετρικῶς
- 420.2 ἐπιτρίτ {ras.} 19–22 legi nequit
- 267.6 ἐπίτριτον 7–13 legi nequit 14 lac. (ἐπόγδοον)
- 268.9 θεωρεῖσθαι 10 λήμματος 11 ἐπιτείνουσι — φιβ ras. 13 αὐτὸν τὸν φιβ περιέχει αὐτόν τε καὶ τὸν ὄγδοον 17 ἀπὸ χμη 18 τὸν ψκθ
- 269.1 post ἐπειδὴ ras. 2 ras. (ὄγδοον) 5–6 τόνος, {ras.} τόνος· οὐδὲ 8 ἐπὶ τοῦ διὰ 9 λήμματος 14 τοῦ σι {ras.} καὶ τῷ κζ 16 ἀποτίζειν 19 {ras.} περιέχεται {ras.} ὑπὸ
- 270.1 τὸ λβ 2 διάστημα om. 3 τοῦ<sup>1</sup> om. 6 ἡμισ {ras.} 8 ἐπογδόφ ἀριθμὸς 10 τῶν οβ 14 λήμματος καὶ δύο τόνων {ras.} τὸ 17 τὸν ,ασφς
- 271.1 τόνον καὶ ποιουσί {ras.} ρη 2 τοῦ ωξδ 4–5 ,ασφς — ,ασφς ras. 6 {ras.} ἐστὶν ὄγδοος 8 ,ακδ 10 δοος τοῦ ,ακδ. {ras.} τοίνυν 11 om. τόνος 13 ἀριθμοὺς 14 οὔτε τοῦ {ras.}· ἐν



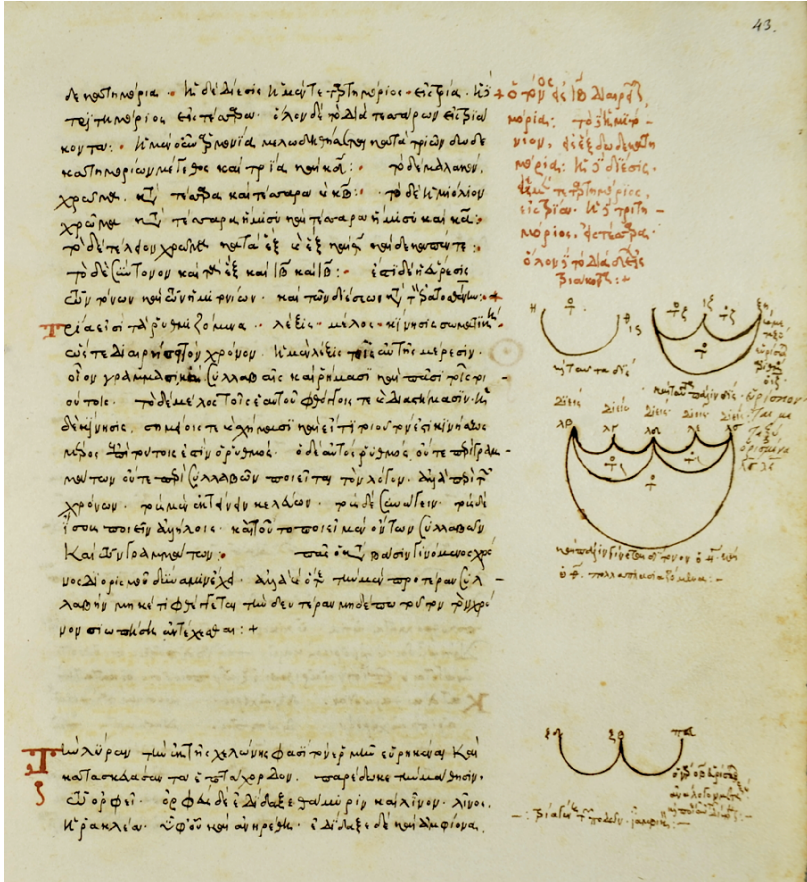


Figure 1. Napoli, Biblioteca Nazionale Centrale Vittorio Emanuele III, III.C.2, f. 43r. Su concessione del MiBACT. È vietata ogni ulteriore riproduzione con qualsiasi mezzo.

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