Unaccountable Numbers

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In memoriam Alessandro Lami,
a tempi migliori

THE AIM of this article is to discuss and amend one of the most intriguing *loci corrupti* of the Greek mathematical *corpus*: the definition of the "unknown" in Diophantus' *Arithmetica*. To do so, I first expound in detail the peculiar terminology that Diophantus employs in his treatise, as well as the notation associated with it (section 1). Sections 2 and 3 present the textual problem and discuss past attempts to deal with it; special attention will be paid to a paraphrase contained in a letter of Michael Psellus. The emendation I propose (section 4) is shown to be supported by a crucial, and hitherto unnoticed, piece of manuscript evidence and by the meaning and usage in non-mathematical writings of an adjective that in Greek mathematical treatises other than the *Arithmetica* is a sharply-defined technical term: ἄλογος. Section 5 offers some complements on the Diophantine sign for the "unknown."

1. *Denominations, signs, and abbreviations of mathematical objects in the* Arithmetica

Diophantus' *Arithmetica* is a collection of arithmetical problems:[1] to find numbers which satisfy the specific constraints that

[1] "Arithmetic" is the ancient denomination of our "number theory." The discipline explaining how to calculate with particular, possibly non-integer, numbers was called in Late Antiquity "logistic"; the first explicit statement of this separation is found in the sixth-century Neoplatonic philosopher and mathematical commentator Eutocius (*In sph. cyl.* 2.4, in *Archimedis opera* III 120.28–30 Heiberg): according to him, dividing the unit does not pertain to arithmetic but to logistic. An earlier definition of logistic, most likely to be ascribed to Geminus (a 1st cent. B.C. mathematically-minded philosopher

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are stated in the enunciation of the problem itself.[2] For instance, *Arithm.* 1.30 requires to find two numbers such that their difference and their product are given numbers. Each problem of the *Arithmetica* is solved by concretely assigning the given numbers, positing one unknown, and solving the equality ("equation" in our language) resulting from the constraints stipulated in the enunciation. In the case of *Arithm.* 1.30, the given numbers are assigned to be 4 and 96; therefore, the constraints stipulated in the enunciation are that the difference and the product of the numbers to be found are 4 and 96, respectively; the procedure of solution gives 12 and 8 as the outcome.[3]

At the beginning of his treatise, Diophantus explains the notation that he will use throughout; he is the first Greek mathematician who consistently adopts a set of signs in order to make his text more concise and, in a sense, conducive to the kind of "algebraic" manipulations forming the technical core of his method for solving numerical problems. In particular, he establishes a terminology to denote what in algebraic language

and polymath, maybe a pupil of Posidonius), does not allow dividing the unit; this definition can be read at ps.-Hero *Def.* 135.5–6 (*Heronis opera* IV 98.12–100.3 Heiberg) and, in a fuller form, as a scholium to Pl. *Chrm.* 165E6 (schol. 27, p.173 Cufalo); echoes of this limitation persist in Domninus *Ench.* 15, p.110.16 Riedlberger (rightly corrected from λογικῆς to λογιστικῆς). It is likely that the domain of logistic was enlarged to include fractional parts as a (later) consequence of the adoption of the sexagesimal system in Greek mathematical astronomy, sometime about Hipparchus' life span, which certainly included the interval 147–127 B.C.

[2] The Diophantine writings were edited by P. Tannery, *Diophanti Alexandrini opera omnia* I–II (Leipzig 1893 text and transl., 1895 *Pseudepigrapha*, *testimonia*, *scholia*, *index graecitatis*). A new edition of the *Arithmetica* has been provided in A. Allard, *Diophante d'Alexandrie*, *Les Arithmétiques* I–II (diss. Louvain 1980, unpublished). The *Arithmetica* was paraphrased in English and commented on extensively in T. L. Heath, *Diophantus of Alexandria. A Study in the History of Greek Algebra* (Cambridge 1910).

[3] It is simple to check that $12 - 8 = 4$ and $12 \times 8 = 96$: therefore the difference and the product of 12 and 8 are the assigned numbers 4 and 96. Of course, the procedure of solution adopted in the *Arithmetica* does not coincide with this *a posteriori* check.

are the powers of the "unknown" x^2, x^3, …; in Diophantus' theoretical framework, these are abstract numerical εἴδη "species," namely generic square, cube, … numbers. The species introduced are assigned a denomination and a conventional sign; the sign is made of the first letter of each component of the denomination, possibly supplemented with the second letter (this always happens to be *upsilon*): to the generic square number (δύναμις) corresponds the sign Δ^{Y}, to the κύβος the sign K^{Y}, to the fourth power (δυναμοδύναμις) the sign $\Delta^{Y}\Delta$, etc.[4] These species must not be confused with particular numbers that happen to be square, cube, fourth powers…[5] On a

[4] See I 2.14–6.2 Tannery. Capital Δ, K, and Y are currently printed, but of course no indication to that effect is contained in the text. It is quite obvious that our notation owes very much, both in conception and in the form of the signs, to Diophantus': note his use of the term δύναμις "power" and the idea of putting a part of the conventional sign "at the exponent." One crucial difference is that we conceive of the species as powers of the "unknown," whereas Diophantus draws a sharp distinction between these notions, as we shall see presently. This difference is made particularly conspicuous by the fact that Diophantus' conventional signs all have the same exponent (the insignificant letter *upsilon*) and a variable "base" indicating the species (letters Δ and K, possibly doubled), whereas modern algebraic signs all have the same base (the most significant "unknown" x) and a variable exponent indicating the power to which the base is to be raised.

[5] Diophantus highlights this difference when he alludes to the Euclidean definition of number (*Elem.* 7.def.2) and when he defines a square number: in both cases he adds a τινος, either to πλήθους or to ἀριθμοῦ. This means that the object so qualified is particular, yet generic (cf. I 2.15 and 2.18; the former passage is quoted in n.21 below, the latter states that square numbers οἵ εἰσιν ἐξ ἀριθμοῦ τινος ἐφ' ἑαυτὸν πολυπλασιασθέντος, "are those 'resulting' from a certain number multiplied by itself"). The following considerations may help further clarify the point. Diophantine numerical species were invoked by the fourth-century mathematical polymath and commentator Theon of Alexandria (*In Alm.* 452.21–453.16 Rome) to explain the structure of orders within the sexagesimal system used by the astronomers. The sexagesimal orders are in fact numerical εἴδη; they correspond to the orders of magnitude in the decimal system: hundreds and thousands are numerical εἴδη, since they are squares (the "unit" of the "hundreds," namely 100, is the square of 10) and cubes (1000 is the cube of 10), respectively; these numerical species do not coincide with particular

terminological level, Diophantus settles the problem of separating particular square numbers from the species "square" by means of the opposition τετράγωνος/δύναμις; a lexical ambiguity (admittedly quite harmless) remains in the case of the κύβος, which may designate both a particular cube number (such as 8) and the species "cube."[6] In order to forestall such ambiguities, I shall refer to the Diophantine species with the denominations "2-species," "3-species," etc.[7]

At the *end* of the list of species, Diophantus also assigns a denomination and a conventional sign to the most generic abstract number, namely one that neither is a particular number nor can be said to have the features characterizing one of the aforementioned species;[8] I shall call it, with a slight abuse of language,[9] the "1-species"; it corresponds to the "unknown" of

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numbers: indeed, 300 is not a square, but 3 items of the square εἶδος "hundreds"; conversely, the species "hundreds" is not a number (it does not even coincide with number 100). This also holds true for fractional numbers: the "seconds" of the sexagesimal system belong to the species "square," insofar as 1/3600 is the square of 1/60.

[6] Apparently, Diophantus did not distinguish between denominations of particular numbers and of species in the case of "powers" higher than the cube, either. This confusion is *totally* harmless, since Diophantus never mentions again in his treatise either species higher than the cube or particular numbers insofar as they happen to be higher powers, such as, for instance, 16 insofar as it is the fourth power of 2.

[7] Note that the species are not mutually exclusive; for instance, any 4-species is also a 2-species: every fourth power is also a square (see also n.9 below).

[8] To repeat: this is not a definition of number (that was provided at I 2.14–15 by alluding to the Euclidean definition), but a definition of a well-defined numerical species. Note too that, in Greek arithmetics, the unit is not a number.

[9] The abuse of language lies in the fact that my denomination "1-species," while formed in exactly the same way as the denominations of the higher species, corresponds to an abstract numerical object that is not defined by Diophantus in the same way as the higher species are—on the contrary, it is defined by negation of the logical sum of the *definientes* of the other species: ὁ δὲ μηδὲν τούτων τῶν ἰδιωμάτων κτησάμενος. Among other

present-day algebra. Let us read this crucial definition, which will be identified henceforth as "the Diophantine sentence," in the Greek text printed in Tannery's edition:[10]

> ὁ δὲ μηδὲν τούτων τῶν ἰδιωμάτων κτησάμενος, ἔχων δὲ ἐν ἑαυτῷ πλῆθος μονάδων ἀόριστον, ἀριθμὸς καλεῖται καὶ ἔστιν αὐτοῦ σημεῖον τὸ ς.

Let us also read Tannery's Latin translation, and the English version by Heath:[11]

> Qui vero nullam talem proprietatem possidet, continet autem in seipso quantitatem unitatum indeterminatam, vocatur *arithmus* [incognitus] et huius signum est ς [*x*].
>
> But the number which has none of these characteristics, but merely has in it an indeterminate multitude of units, is called ἀριθμός, 'number', and its sign is ς [= *x*].

Since all enunciations of problems in the *Arithmetica* require to find (particular) ἀριθμοί under assigned conditions, the terminological choice ἀριθμός for the 1-species is far more unfortunate than keeping to the denomination κύβος both for a particular cube number and for the 3-species;[12] apparently,

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things, this entails that no *n*-species is also a 1-species (see n.7 above). If species were to be identified with particular numbers, the text we are about to read would have singled out quite a weird class of numbers: those that are not powers (2, 3, 5, 6, 7, 10, …).

[10] At I 6.3–5. Note the masculine article at the beginning: Diophantus introduces each species by directly calling it ἀριθμός "number," a fact that provides a decidedly tautological turn to the Diophantine sentence; the denomination εἶδος will first appear at I 6.21, after "inverse species" are introduced (see n.33 below), and will feature consistently throughout the outline of the method for solving numerical problems at I 14.1–20.

[11] At I 7 and Heath, *Diophantus* 130, respectively.

[12] The point can be clarified by looking at *Arithm*. 1.1. The beginning of this problem reads (μº is the sign Diophantus prescribes for the μονάς; it can only accompany particular [ὡρισμένοι] numbers, see I 6.6–8): τὸν ἐπιταχθέντα ἀριθμὸν διελεῖν εἰς δύο ἀριθμοὺς ἐν ὑπεροχῇ τῇ δοθείσῃ. ἔστω δὴ ὁ δοθεὶς ἀριθμὸς ὁ ρ, ἡ δὲ ὑπεροχὴ μº μ. εὑρεῖν τοὺς ἀριθμούς. τετάχθω ὁ ἐλάσσων ς α (I 16.9–13), "To divide an assigned number into two numbers in a given difference. Then, let the given number be 100, the difference 40

and as Diophantus himself expressly states before presenting the species,[13] his system of conventional signs was intended to be used consistently and throughout all problems of the treatise. In order to avoid confusions I shall always write "ἀριθμός" (without the determinative article) when referring to a particular number, "the ἀριθμός" when referring to the 1-species.

2. *An intriguing locus corruptus*

The problem with the Diophantine sentence is that it contains one of the most intriguing *loci corrupti* offered by Greek mathematical texts.

To see this, and because Tannery's apparatus is notoriously unreliable, let us turn to the readings of the manuscripts. The rich tradition of the *Arithmetica* (31 witnesses) can readily be reduced to four independent sources: *Matrit.* 4678,[14] *Vat.gr.*

u(nits). To find the numbers. Let the lesser [number] be set to be 1*x*." The assigned number, later given as 100, is called ἀριθμός, the sought numbers (particular but unknown until the end of the problem) are called ἀριθμοί, the "1-species" is denoted by the sign for the ἀριθμός, even if all manuscripts (wrongly) write ἀριθμοῦ ἑνός instead of ς α (to wit, "number one" instead of "1*x*"). Another source of confusion is the sign that Diophantus introduces for the ἀριθμός; I shall deal with the issue in the Complement at the end of this paper.

[13] At I 4.12–14: ἐδοκιμάσθη οὖν ἕκαστος τούτων τῶν ἀριθμῶν συντομωτέραν ἐπωνυμίαν κτησάμενος στοιχεῖον τῆς ἀριθμητικῆς θεωρίας εἶναι, "Now, each of these numbers, once it has got an abbreviated denomination, is fit to be an element of arithmetic theory." The "elements of arithmetic theory" are the species whose denominations and signs Diophantus is about to introduce, the ἀριθμοί referred to are the kinds of particular numbers just described (squares, cubes, …), whose denominations are adopted, with the sole exception of τετράγωνος, as the denominations of the species themselves. Thus, the operation of assigning an "abbreviated denomination" transforms numbers into numerical species. See also n.10 above.

[14] This manuscript contains Nicomachus *Introductio arithmetica* (ff. 4r–57v), Diophantus *Arithmetica* and *De polygonis numeris* (58r–130v and 130v–135v), Cleonides/[Euclid] *Introductio harmonica* (137r–142r), Euclid *Sectio canonis* (142r–143v, incomplete). I. Pérez Martín, "Maxime Planude et le *Diophantus Matritensis* (*Madrid, Biblioteca Nacional*, ms. 4678): un paradigme de la récupération des textes anciens dans la 'renaissance paléologue'," *Byzantion* 76

191,[15] *Vat.gr.* 304, and *Marc.gr.* 308,[16] this last in fact containing a recension made by the renowned scholar Maximus Planudes (†1305).[17] The texts they present are transcribed below. I have retained almost all of their graphic features, including punctuation; with a few exceptions to be discussed in detail, canonical compendia or abbreviations are expanded with parentheses.

Matrit. 4678, f. 58^{v} (m. 2 = John Chortasmenos):

ὁ δὲ μηδὲν τούτων τ(ῶν) ἰδιωμάτ(ων) κτησάμ(εν)^{o} ἔχων δὲ ἐν αὐτῶι πλῆθος μονάδ(ων), ἄλογος ς^{o}\` καλεῖται ς\` (ἔστιν) αὐτοῦ σημεῖον τὸ ς̄\`·

μηδὲν] μηδ’ ἓν m. 1 sed corr. m. 2 | ς^{o}\`] suprascr. ἀριθμὸς m. 2 | αὐτοῦ σημεῖον] suprascr. (καὶ) ἔστιν αὐτοῦ σ(ημεῖον) τόδε alium signum adcedens m. 2

Vat.gr. 191, f. 360^{r}:

ὁ δὲ μηδὲν τούτων τῶν ἰδιωμάτων κτησάμ(εν)^{o} ἔχων δὲ ἐν αὐτῶ πλῆθος μονάδων, ἄλογ^{o} ’ς^{o}\` καλεῖται καὶ ἔστιν αὐτοῦ σημεῖ(ον) τὸ ς̄.

Vat.gr. 304, f. 77^{r}:

ὁ δὲ μηδὲν τούτων τῶν ἰδιωμάτ(ων) κτησάμ(εν)^{o} ἔχων δὲ ἐν αὐτῶ πλῆθος μονάδων, ἄλογ^{o} ’ς^{o}\` καλεῖται (καὶ) ἔστιν αὐτ(οῦ) σημεῖον τὸ ς̄·

αὐτῷ corr. ex ἑαυτῷ m. 1

(2006) 433–462, presents a detailed paleographic and codicological analysis of this codex, formerly assigned to the thirteenth century, dating it back to the mid-eleventh century.

[15] On this codex, a huge collection written by sixteen copyists between 1296 and 1298, see D. Bianconi, "Libri e mani. Sulla formazione di alcune miscellanee dell'età dei paleologi," *S&T* 2 (2004) 311–363, at 324–333; to one of these copyists we also owe the second part (ff. 56–98) of *Vat.gr.* 203.

[16] *Marc.gr.* 308 was copied at the very end of the thirteenth century, *Vat.gr.* 304 displays watermarks dated to the second and third decade of the fourteenth century.

[17] The most recent analysis of the manuscript tradition of the *Arithmetica* was provided by A. Allard, "La tradition du texte grec des *Arithmétiques* de Diophante d'Alexandrie," *RHT* 12–13 (1982–1983) 57–137.

Marc.gr. 308, f. 52v:

> ὁ δὲ μηδὲν τούτων τῶν ἰδιωμάτων κτησάμενος. ἔχων δὲ ἐν ἑαυτῶ πλῆθος μονάδων, ἄλογος ἀριθμὸ καλεῖται, καὶ ἔστιν αὐτοῦ σημεῖον τὸ ς̄`.

It is fairly obvious that the manuscript tradition hands down one and the same text to us. Accordingly, in his edition Allard prints the following text and translation:[18]

> ὁ δὲ μηδὲν τούτων τῶν ἰδιωμάτων κτησάμενος, ἔχων δὲ ἐν ἑαυτῷ πλῆθος μονάδων, ἄλογος ἀριθμὸς καλεῖται, καὶ ἔστιν αὐτοῦ σημεῖον τὸ ς.

> Le nombre qui n'a reçu aucune des caractéristiques précédentes, mais qui contient une certaine quantité d'unités, s'appelle nombre provisoirement non déterminé, et son symbole est ς.

Tannery's emendation is a bold one: he shifted the comma and replaced ἄλογος with ἀόριστον, making it a modifier of πλῆθος and not of ἀριθμός. Yet, some correction is required: it is quite obvious that, *pace* Allard,[19] the Diophantine sentence as transmitted by the manuscripts cannot stand. First, one should print in the critical text αὑτῷ and not ἑαυτῷ, since the former is the most economical emendation of the readings of the manuscripts.[20] Second, and most important, a determinative of πλῆθος in the participial clause ἔχων δὲ ἐν αὑτῷ πλῆθος μονάδων is necessary, both syntactically and semantically.[21] Third, a

[18] *Diophante* 375.11–13 and 424, respectively. In his apparatus Allard also does not report correctly the readings of the manuscripts (see n.20 below).

[19] That the text cannot be sound is already shown by the translation proposed by Allard: he must introduce "certaine" as a most needed determinative of πλῆθος; he unduly adds "provisoirement" to the questionable translation of ἄλογος "non déterminé" (this would more properly be a translation of Tannery's ἀόριστος). Note also the incongruous "symbole" for what is in fact a "sign."

[20] Tannery only reports the variant readings of the *Matritensis*; Allard (apparatus at *Diophante* 411) wrongly ascribes the reading ἑαυτῷ to all the other three witnesses.

[21] Compare (n.5 above) the presence of τινός in the clause at I 2.14–15 πάντας τοὺς ἀριθμοὺς συγκειμένους ἐκ μονάδων πλήθους τινός (a modi-

determinative of the subsequent ἀριθμός, let it be ἄλογος or whatever else, would simply be useless in a conventional designation of the most basic entity in a series. Fourth, the link between ἄλογος and ἀριθμός that the manuscripts unanimously attest is quite straightforwardly contradicted (a) by remarking that a two-word designation within a series of one-word designations would sound very odd,[22] and (b) by the fact that, in the preface of the *Arithmetica*,[23] the "1-species" is always designated by ἀριθμός, not by ἄλογος ἀριθμός. Fifth, and this probably is what Tannery mainly had in mind, an ἄλογος ἀριθμός is a *contradictio in adjecto*: ἄλογος is a well-established technical term of Greek mathematics and means "irrational" (see below)—and an integer or fractional number, as any solution of a Diophantine problem must be, can by no means be "irrational."

3. *Getting help from Michael Psellus: alternative denominations of numerical species*

No help in amending the text comes from the scholia to the *Arithmetica*, nor from the extensive paraphrase of the introduction of the Diophantine treatise that was redacted by George Pachymeres (b. 1242) in his *Quadrivium*: his text is identical with the one printed by Allard, without the final clause καὶ ἔστιν αὐτοῦ σημεῖον τὸ ς.[24]

A look at Tannery's apparatus shows that he drew his correction from a previously unpublished letter of Michael Psellus

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fication to a participial clause of *Elem.* 7.def.2 ἀριθμὸς δὲ τὸ ἐκ μονάδων συγκείμενον πλῆθος, in which the determinative of πλῆθος is τὸ ἐκ μονάδων συγκείμενον), whose structure is similar to that of ἔχων δὲ ἐν αὐτῷ πλῆθος μονάδων.

[22] One must not forget that Diophantus resorted to one-word wild coinages such as δυναμόκυβος.

[23] That is, in I 2.3–16.7. As explained in nn.12–13 above, every occurrence of the ἀριθμός in the series of problems should be written as the sign ς̄ (macron included).

[24] See P. Tannery and E. Stéphanou, *Quadrivium de Georges Pachymère* (Vatican City 1940) 46.5–6. Note that we read this treatise in an autograph of its author: it is the codex Rome, *Biblioteca Angelica gr.* 38 (see *RGK* III 115).

(b. 1018), in which the renowned Byzantine scholar and polymath explains to his anonymous addressee some basic notions and tools of number theory and metrology: the denominations of the numerical species[25] and their usefulness in solving arithmetic riddles in the form of epigrams; how to measure a number of simple solids. Psellus finally mentions the arithmological lucubrations contained in the so-called "letter of Petosiris to Nechepson" and in the "little Pythagorean plinth," just to declare that they are a heap of nonsense.[26] When he comes to introduce the ἀριθμός, Psellus offers the following paraphrase of the Diophantine sentence:

> ἀριθμὸς δὲ παρ' αὐτοῖς ἰδιαίτερον λέγεται ὁ μηδὲν μὲν ἰδίωμα κτησάμενος, ἔχων δὲ ἐν ἑαυτῷ πλῆθος μονάδων ἀόριστον· καλεῖται δὲ αὐτοῖς οὗτος ὁ ἀριθμὸς καὶ πλευρά.

If we are to believe the text and the apparatus of Tannery's edition of Psellus' letter, the structure of this sentence gets rid of the ambiguity in the corresponding sentence in Diophantus, in

[25] Psellus calls this "the Egyptian method" simply because his sources, Diophantus and Anatolius, were both based in Alexandria—nothing to do with early Egyptian arithmetic.

[26] The letter was first partly published by Tannery himself, "Psellus sur Diophante," *Zeitschrift für Mathematik und Physik. Historisch-literarische Abt.* 37 (1892) 41–45 (repr. *Mémoires scientifiques* IV [Toulouse/Paris 1920] 275–282), at 42–43 (277–278), and in its complete form at *Diophanti opera* II 37–42: see 37.3–39.10 for the part pertaining to number theory, 37.10–13 for the quotation (metrological issues are addressed at 39.11–41.21, arithmology is liquidated at 41.22–42.13). The letter is attested in the following MSS.: *Scorial.* Y.III.12, ff. 73ʳ–74ᵛ, *Laur.Plut.* 58.29, ff. 196ʳ–197ʳ (which I have checked for the text), *Vat.Urb.gr.* 78, f. 81ʳ⁻ᵛ; see P. Moore, *Iter Psellianum* (Toronto 2005) 311, item PHI.158 [881]. For indications on the former of the two Neopythagorean texts see E. Riess, "Nechepsonis et Petosiridis fragmenta magica," *Philologus* Suppl. 6 (1891–1893) 325–394, at 387 (nos. 41–42); for an edition of the latter see P. Tannery, "Notice sur des fragments d'onomatomancie arithmétique," *Notices et extraits des manuscrits de la Bibliothèque Nationale* 31.2 (1886) 231–260 (repr. *Mémoires scientifiques* IX [Toulouse/Paris 1929] 17–50). An analysis of the mathematics behind such writings is in O. Neugebauer and G. Saliba, "On Greek Numerology," *Centaurus* 31 (1989) 189–206.

which ἄλογος is placed just between πλῆθος μονάδων and ἀριθμός; Tannery simply adopted Psellus' text as if it were a transcription of the "original" Diophantine sentence.[27]

Now, it is in my opinion clear what Psellus' (or one of his sources') *varia lectio* amounts to: it is simply a semantic *lectio facilior*, at the same time trivializing the quoted text and explicative of it; after all, Psellus' intent was to explain Diophantus' notation to his addressee. As a consequence, one is not entitled to amend the Diophantine sentence, as Tannery does, by simply replacing the crucial word ἄλογος with its gloss. On the other hand, exactly because of Psellus' intent, his paraphrase provides us with crucial indications as to the structure of the original: the comma in the manuscripts must be misplaced; the word necessarily replacing the corrupt ἄλογος must qualify πλῆθος and not ἀριθμός. Most importantly, Psellus tells us that such an amended word must remain in the semantic domain of indeterminacy;[28] the term ἀόριστον he chose in order to gloss ἄλογο* shows his lexical skills: *alpha* privative as in ἄλογο*, λόγος and ὁρισμός sharing a currently used meaning, namely "definition."

It is, I think, by now quite clear how we should correct the passage of the *Arithmetica*. However, a discussion of the main features of the system expounded by Psellus, in fact an enriched version of Diophantus', will add important clues to our dossier.

First, as Psellus himself declares,[29] he quite surely resorted to

[27] Tannery held that Psellus had drawn his exposition of the numerical species from scholia to a manuscript of the *Arithmetica*; from the same scholia the adjective ἄλογος (there qualifying the δυναμόκυβος, see below) crept into the text and replaced the original ἀόριστον: *Zeitschrift für Mathematik und Physik* 37 (1892) 42 (repr. 276–277), and *Diophanti opera* II IX–X.

[28] Cf. again nn.5 and 21 above. Even if Psellus was very likely still living when the *Matritensis* was transcribed (Pérez Martín, *Maxime Planude* 439–441), I am fairly sure that he read a sound version of the Diophantine sentence—or at least a version in which the ambiguities due to compendia were not settled on a wrong text. For this reason, I shall occasionally use the partially undetermined ἄλογο*.

[29] Psellus' reference to Anatolius reads: περὶ δὲ τῆς αἰγυπτιακῆς μεθόδου

the popularization of Diophantus' notation authored by some λογιώτατος Anatolius, maybe to be identified with the person whose name is attached to a treatise on the Decade, a specimen of the literary sub-genre of *theologumena arithmeticae*.[30] It is an easy guess that our passage was made *facilius*, by introducing

ταύτης Διόφαντος μὲν διέλαβεν ἀκριβέστερον, ὁ δὲ λογιώτατος Ἀνατόλιος τὰ συνεκτικώτατα μέρη τῆς κατ' ἐκεῖνον ἐπιστήμης ἀπολεξάμενος ἑτέρω Διοφάντῳ συνοπτικώτατα προσεφώνησε (II 38.22–39.1). There are two problems in this sentence. First, Tannery suspected συνοπτικώτατα to be a dittography of συνεκτικώτατα, but the difference between "most essential" and "in a most succint way" exactly fits both the meaning of the sentence and the features of the system that we read in Psellus. The second problem lies in the word I have left written ἑτέρω. According to Tannery (II 38.25 *in app*.), this is the reading of the manuscripts. He therefore suspected a scribal mistake, not simply the usual omission of mute *iota*. Accordingly, he corrected to ἑτέρως, but in the *prolegomena* to the edition he recanted and suggested to correct to "ἑταίρῳ vel <τῷ> ἑταίρῳ" (II XLVII). W. R. Knorr, "*Arithmêtikê stoikeiôsis*: On Diophantus and Hero of Alexandria," *HM* 20 (1993) 180–192, at 184, proposed an obvious emendation: restore the mute iota (in fact, this is the reading of *Laur.Plut*. 58.29, f. 196r, fifth line from bottom: the subscript iota is quite conspicuous) and postulate that two different Diophantus are at issue: the mathematician and the addressee of Anatolius' synopsis. Admittedly, this coincidence is quite unlikely, and one wonders why Psellus would find giving *his* addressee the information of the name of the addressee of Anatolius' synopsis so interesting (in Knorr's article, the hypothesis serves to [allegedly] corroborate his thesis that the author of the pseudo-Heronian *Definitiones* is in fact Diophantus—the mathematician, not Anatolius' addressee). Another possibility is to keep Tannery's ἑτέρως in the text and correct Διοφάντῳ to Διοφάντου ("in a different way from Diophantus'")—but then to whom was Anatolius' synopsis addressed?

[30] A good introduction to the several Anatolius living ca. the third century CE is R. Goulet, *DictPhilAnt* I (1989) 179–183; the edition of the arithmological tract ascribed to one Anatolius (amply excerpted in the pseudo-Iamblichean *Theologumena*) is in J. L. Heiberg, "Anatolius sur les dix premiers nombres," *Annales internationales d'histoire, Congrès de Paris 1900, 5e section, Histoire des sciences* (Paris 1901) 27–57; on the stemmatic structure of the entire tradition of Greek arithmological writings see F. E. Robbins, "The Tradition of Greek Arithmology," *CP* 16 (1921) 97–123.

the gloss ἀόριστον, already in Anatolius' popularization.[31]

Second, the numerical species are presented by Psellus in inverse order with respect to that adopted by Diophantus: μονάς → the ἀριθμός → higher species. In this way, however, Psellus' characterization quoted above amounts to a definition of "number," and in fact to a severe distortion of the Euclidean definition; it is not a definition of the 1-species. This is the reason why Psellus' characterization has a quite contrived look: the term ἰδίωμα, once the demonstrative τούτων in the Diophantine sentence is eliminated, remains without a *relatum*; the article preceding the second occurrence of ἀριθμός is unnecessary. All of this undermines the rationale behind Diophantus' exposition.

Third, after the text quoted above, Psellus sets out to describe the several species, but exemplifies them with particular numbers (he uses the powers of 2), whereas we have seen that Diophantus crucially distinguishes particular numbers from species.[32]

Fourth, the denominations are extended to higher species than in the *Arithmetica*, where the last species introduced is the κυβόκυβος (6-species). Psellus goes as far as the 9-species, even if for the inverse species he stops, exactly as Diophantus did, at the κυβοκυβοστόν.[33]

[31] That this was the case is suggested by the fact that the Diophantine sequence of numerical species, from ἀριθμός, μονάς, δύναμις (note the order) up to κυβόκυβος, is presented as standard Pythagorean lore in Hippolytus *Ref.* 1.2.6-10 (repeated at 4.51.4-8). Most notably, ἀριθμός is made the common genus of all numbers, including the subsequent species; as such, it is twice called ἀόριστος. Hippolytus' short exposition contains a number of inconsistencies; I take it as certain that it is an unsuccessful attempt to graft Diophantus' system onto Pythagorean doctrine.

[32] At II 37.13–38.15. But one must admit that the way Diophantus plays with the word ἀριθμός (n.12 above) does not help understanding his subtle distinctions.

[33] The inverse species are related to the species exactly as parts are related to numbers: as ⅓ is the inverse of 3, so the δυναμοστόν is the inverse of the δύναμις (I 6.9–19).

Fifth, all species are given alternative names, according to their rank: ἀριθμός = ἀριθμὸς πρῶτος, δύναμις = ἀριθμὸς δεύτερος, …; again, some species starting from the fifth are given further alternative names: 5-species (= δυναμόκυβος = ἀριθμὸς πέμπτος) = ἄλογος πρῶτος, 7-species (= ἀριθμὸς ἕβδομος) = ἄλογος δεύτερος, 8-species = τετραπλῆ δύναμις, 9-species = κύβος ἐξελικτός.[34] These denominations are descriptive, with one notable exception: the two ἄλογοι. Note that here ἄλογος is treated as a substantive.

Sixth, here is the inconsistent (or incomplete, or both) explanation that Psellus offers of the denomination ἄλογος πρῶτος: because it is neither a square nor a cube.[35] This shows what some readers of Diophantus felt entitled to do with a supposedly technical term like ἄλογος.

The system expounded by Psellus, which he ascribes to Anatolius, taking up "the most essential parts" of Diophantus' doctrine, appears to be a descriptive-classificatory attempt conflating notions and terminology that come from several sources. The idea of adopting the rank within a well-ordered sequence of (mathematical) objects to the effect of creating a "logarithmic" system of denominations (ἀριθμὸς πρῶτος, ἀριθμὸς δεύτερος, …) coincides with that exploited by Archimedes to give names to the several orders of magnitude in the decimal system:[36] and in fact, the denominations are, with two crucial differences that reveal the derivative character of Psellus' clas-

[34] That is, "revolved cube," since its sides are also cubes. In the same way, the 4-species might also have been called δύναμις ἐξελικτή. The adjective is not attested in LSJ, nor have I found occurrences in the TLG.

[35] At II 38.2–3: οὔτε γὰρ τετράγωνός ἐστιν οὔτε κύβος. The explanation is flawed since it refers to the 5-species but it applies to the 7-species as well. Psellus should have at least specified that his explanation only has scope over the genus ἄλογος.

[36] The system is described in the *Arenarius* (*Archimedis opera* II 236.17–240.19 Heiberg, with an additional lemma at II 240.19–242.19). The trick of converting ranks to denominations is applied recursively by Archimedes, by simply changing the ordered sequence of reference.

sification,[37] identical with those introduced by Archimedes. As for the other denominations, the micro-system of ἄλογοι included, the likely identification of Psellus' Anatolius with the author of the tract on the Decade might suggest a Neopythagorean origin, even if the lexicon employed does not specially recommend this option: no occurrence of ἄλογος in a similar sense and in technical contexts can be found in the writings of Nicomachus or of Iamblichus.[38]

4. *Amending the Diophantine sentence*

The right, and at any rate most economical, way to amend the Diophantine sentence is quite obvious: correct ἄλογος to ἄλογον and shift the comma after it, the comma's position before ἄλογος in our manuscripts having been induced by the fact that ἄλογος in the nominative can only go with the subsequent ἀριθμός.[39] The result is: … ἔχων δὲ ἐν αὑτῷ πλῆθος μονάδων ἄλογον, ἀριθμὸς καλεῖται …

From the paleographic point of view, the problem of justifying the change in termination from -ον to -ος is straightforwardly dealt with by noting that supralinear *omicron* was, even in late Byzantine manuscripts, also a mark of abbreviation

[37] The first difference is purely mathematical: Archimedes' ἀριθμοὶ πρῶτοι range from the unit to the decimal 8-species (myriad of myriads) excluded, the ἀριθμοὶ δεύτεροι from one myriad of myriads, taken as the new unit, to the decimal 16-species excluded, … The second difference is that Archimedes' denominations refer to classes of particular numbers, and hence are only used in the plural: there does not exist a species called ἀριθμὸς δεύτερος, but a set of particular ἀριθμοὶ δεύτεροι.

[38] Just two occurrences in these authors have a technical meaning. At Nicomachus *Intr.Arith.* 1.6.3, entities ἄλογα πρὸς ἄλληλα are opposed to those λόγον πρὸς ἄλληλα ἔχοντα "having a ratio to each other"; the context is that of a general discussion of the concept of number. At Iamblichus *In Nic.* 4.146 (160.29–30 Vinel = 91.13–14 Pistelli), it is asserted that any one of the side and the diagonal of a square is ἄλογος whenever the other is assigned a rational value (this statement is at variance with the theory of irrational lines expounded in Euc. *Elem.* 10, see below).

[39] The comma is quite vigorously marked in the *Matritensis*, but to this key feature I shall return below.

by suspension, and not only the sign for the termination -ος.[40] The change could even have occurred at a very early stage of transmission, since compendia for terminations are quite systematically absent in early majuscule or minuscule codices,[41]

[40] See most recently L. Tarán, "The Text of Simplicius's Commentary on Aristotle's *Physics* and the Question of Supralinear Omicron in Greek Manuscripts," *RHT* 9 (2014) 351–358. To the examples and to the references to standard paleographic textbooks adduced by Tarán, we may add the occurrences of supralinear *omicron* as an abbreviation of -ον at *Alm*. 6.9 and 11.6, recorded in the critical apparatus at *Ptolemaei opera* I.1 527.1 and I.2 414.7 Heiberg, respectively (the manuscript involved in both instances is *Vat.gr*. 180). Heiberg calls this and other non-standard compendia "uestigia antiquioris tachygraphiae" (*Ptolemaei opera* II LXXXIX). That Heiberg was right is confirmed by a manuscript penned by Ephrem, namely *Athen*. 1 (Theodoret of Cyprus *Comm. in Psalmos*): the two occurrences of supralinear *omicron* as an abbreviation of -ον at f. 213^{r} lines 5 and 15 occur in passages of the Psalms commented on and therefore written in majuscule; see plate 4 of G. Prato, "Il monaco Efrem e la sua scrittura," *S&C* 6 (1982) 99–115. That supralinear *omicron* can be a generic sign of abbreviation by suspension comes as no surprise for anyone acquainted with Greek arithmetical, astronomical, or logistic texts: the sign for μονάς or for μοῖρα is very frequently μ^{o}, provided with no termination (cf. n.12 above; on the syntactical problems raised by this practice, surely dating back to the originals and strictly adhered to by all copyists, see Rome's remarks, *Theon In Alm.* XXIV–XXVI). Supralinear *omicron* was not the only generic sign of abbreviation by suspension that later became the standard compendium of a specific termination, as Heiberg explains at *Ptolemaei opera* II XCI, a passage that deserves to be quoted in full: "omnino ratio abbreuiandi adscripta nota ' uel ς ideo saepius errandi occasionem dedit, quod librarii posteriores eius ignari lineolam pro compendio aliquo tachygraphico accipiebant syllabam certam repraesentante, cum nihil nisi abbreuiationem in uniuersum significaret ex sententia supplendam; uelut ς saepissime non ης significat, sed quamlibet terminationem, [*a 43-item list follows*], item ` non ον, sed lineolam abbreviationis, [*a 35-item list follows*]."

[41] For mathematical texts, it suffices to check the *fragmentum mathematicum bobiense*. This is the *scriptio inferior*, dated to the 5^{th}–6^{th} century, of the palimpsest *Ambros*. L 99 sup. (Isidore of Seville *Etymologiae*), whose pages 113–114 are reproduced in C. Belger, "Ein neues Fragmentum mathematicum Bobiense," *Hermes* 16 (1881) 261–284. The phenomenon of absence of terminations is particularly conspicuous in the case of substantives designating mathematical objects, such as γωνία, πλευρά, etc.

and at any rate such compendia may easily remain in the pen.

A striking feature of the manuscript tradition strongly corroborates my hypothesis.[42] In *Matrit.* 4678, f. 58^v line 8, the final *sigma* of ἄλογος is in fact the result of the correction of a *nu*; the comma following μονάδων is so strongly marked (there even are two commas, combining to produce a sort of very distorted *nu*, which however must not be taken as the final *nu* of μονάδων) as to make one suspect that it has the function of preparing for the immediately subsequent correction. Thus, the copyist of the *Matritensis* first wrote ἄλογον, maybe because he or an earlier colleague of his had judged attaching the ἄλογ^o he was reading in his model to the preceding πλῆθος to be quite natural, but then corrected himself and also put a comma in the text in order to forestall possible uncertainties as to the form of the final letter of ἄλογος—after all, Diophantus is assigning to a most generic kind of ἀριθμός the denomination "ἀριθμός," which admittedly is quite bewildering. The copyists of *Vat.gr.* 191 and of *Vat.gr.* 304, who also mark a comma after μονάδων and most likely intended their supralinear *omicron* as the compendium for -ος, might have involuntarily "restored" the exact reading of some common ancestor of the entire tradition of the *Arithmetica*. Passages like this, however, make me suspect that such an ancestor simply is the *Matritensis*.[43]

Is just correcting the case-ending of the received ἄλογος really a good solution? What does πλῆθος μονάδων ἄλογον

[42] I thank Dr. M. R. Sanz San Bruno of the Biblioteca Nacional de España for kindly allowing me to examine this fragile codex (accessed 7 May 2015); on my request, I. Pérez Martín confirmed the correctness of my paleographic analysis. A digital reproduction of the codex can be found at http://bdh.bne.es/bnesearch/biblioteca/Diofanto%20de%20Alejandr%C3%ADa (p.128 of the file: the final *sigma* of ἄλογος is the last letter of the line and its form is thereby distorted). However, suspicions as to its being a '*prima intentione*' *sigma* already are raised by looking at the digital reproduction.

[43] *Pace* the stemma proposed by Allard, *La tradition* 76. Tannery (II XXII–XXV) also held that the Madrid codex is the ancestor of the non-Planudean family, but his argument is quite poor.

mean? It is not easy to translate this adjective ἄλογος, whether we attach it to the subsequent ἀριθμός or to the preceding πλῆθος—maybe "undefined" or "undetermined," as in Psellus/Tannery's or in Allard's reading; maybe "unaccountable" as is suggested in the title of the present article:[44] the multiplicity of the units contained in the ἀριθμός cannot be a matter of discourse simply because it is by definition impossible to say what it amounts to. Yet, the difficulty of providing a satisfactory translation of ἄλογος does not mean that bold emendations, as Tannery's is, are required.

It is in fact obvious that in his sentence Diophantus did not intend to use ἄλογος in the strictly technical sense that the term assumes in the theory of irrational lines as expounded in Euclid *Elem*. 10. In this theory, in fact, the adjective qualifies straight lines and regions (hence geometric magnitudes, not numbers) that are incommensurable, in a sense which is ill-suited to represent arithmetical states of affair, with straight lines or regions taken as references.[45] Maybe it is for this reason that, when referring in the *Arithmetica* to solutions that cannot be expressed in numbers (non-rational, in modern parlance), Diophantus never employs ἄλογος, but οὐ ῥητός "non-expressible,"[46] that is, not having to the unit a ratio expressible in

[44] The adjective ἄλογος can also bear a connotation of potentiality, as is easy to verify (LSJ can suffice).

[45] See *Elem*. 10.def.3–4. The Euclidean notion is ill-suited because lines whose squares have to the square on the reference straight line a ratio expressible in numbers, yet not a ratio of square numbers, would not be termed ἄλογοι. In modern parlance, a line that is $\sqrt{2}$ times the reference line is not an "irrational" line in the sense of ἄλογος assumed in *Elem*. 10, but a ῥητή, "expressible": the ratio of the square on this line to the square on the reference line is 2:1; this, of course, is a ratio of a number to a number, still, it is not a ratio of square numbers (for instance, 9:4 is one such ratio).

[46] The fourteen occurrences of the adjective in the *Arithmetica* are distributed as follows: "non-expressible" number, I 204.19, 208.7, 210.1, 212.6–7; "non-expressible" equality (that is, not admitting an expressible solution), 264.13; "non-expressible" double equality (referred to in the neuter), 270.5.

numbers. As a consequence of this lexical choice, the only occurrence of ἄλογος in the *Arithmetica* is in the Diophantine sentence.

Still, even if a term has a well-defined, and canonical, technical sense, it does not follow that one is compelled to take it as a rigid designator, even in technical contexts, and to refrain from using it in more current or metaphoric meanings. It suffices to think of Psellus' two ἄλογοι-species, and of his seemingly sloppy explanation: quite simply, the designations mean that the 5- and 7-species did not have, in the system alternative to the one expounded by Diophantus, a specific denomination —they remain unexpressed or, as it were, unworthy of discourse. All in all, the possibility that Diophantus allowed himself a (in his eyes) harmless wordplay is to be regarded as more likely than not.[47]

On the affirmative side, one has: "expressible" number, I 242.21, 370.5, 400.11, 408.3, 422.13, 430.25, 436.18; "expressible" right triangle (that is, a right triangle whose sides can all be expressed in numbers), I 402.22. Still, this Diophantine terminology is again at variance with the theory of *Elem*. 10, since there, once the reference straight line is fixed, ἄλογος and ῥητός are complementary predicates, so that, *mutatis mutandis*, what is οὐ ῥητός to Diophantus can still be ῥητός *iuxta Elem*. 10 (the example is the same as that in n.45 above). Diophantus was not the only ancient mathematical author who simplified the Euclidean dichotomy "expressible"/"irrational"; for a discussion of the entire documentary record see B. Vitrac, *Euclide*, *Les Eléments* (Paris 1990–2001) III 43–51. In Byzantine logistic treatises, a further terminological shift occurred and ῥητός became synonymous with "integer number"; see for instance the definition of "expressible number" in Theodorus Meliteniotes *Tribiblos astronomike* 1.2 (106.74–76 Leurquin): καί ἐστι ῥητὸς μὲν ἀριθμὸς ὁ ἐκ μοιρῶν μόνων συγκείμενος, ἄρρητος δὲ ὁ μὴ ἐκ μοιρῶν μόνων ἀλλὰ καὶ λεπτῶν συγκείμενος.

[47] There even are a couple of passages in the first two paragraphs of the introduction of the *Arithmetica* in which Diophantus appears to play with his own terminology: cf. the striking phrase <u>ὑποστῆσαι</u> τὴν ἐν τοῖς ἀριθμοῖς φύσιν τε καὶ <u>δύναμιν</u> (I 2.6–7; the play with δύναμις is obvious, the ὑπόστασις is the specific part of a Diophantine problem, in which the numbers to be determined are expressed in terms of the ἀριθμός and possibly of higher species); and the expression προσλαβοῦσα διδαχήν (I 2.13; the verb is a techical term denoting addition).

However, in ancient technical writings one finds less pointed technical meanings of ἄλογος, all obviously related to the main technical meaning of λόγος as "ratio": either a relation between magnitudes otherwise falling in a system of ratios is ἄλογος since it cannot be expressed by a ratio, or the unit of a particular arithmetical system is ἄλογος since it cannot have a ratio to itself.[48]

To the first category belong some specific elaborations of rhythmic and harmonic theory. As for rhythmic theory, Aristoxenus qualifies a foot as ἄλογος whose down-beat is intermediate between twice and once the up-beat; the foot itself is called χορεῖος ἄλογος.[49] The reason for this foot being ἄλογος lies in the fact that the relation between the down-beat and the up-beat is not specified by a well-defined ratio, but the former is only said to lie somewhere between twice and once the latter. The same "deficient" foot (but its name is not given) is evoked by Dionysius of Halicarnassus when he praises the succession of dactyls, "and those filled with ἄλογοι," with which Homer at *Od*. 11.596–598 describes Sisyphus' vain efforts.[50]

In harmonic theory, the second category is represented by a passage in Ptolemy's *Harmonica*, where it is said that "a note is a

[48] See D. Fowler, *The Mathematics of Plato's Academy*[2] (Oxford 1999) 191–193, for a complete list of occurrences of ἄλογος and ἄρρητος/ῥητός in Plato, Aristotle, and the Presocratic philosophers. A discussion of the passages in which these terms assume a technical meaning would bring us too far from the goals of this note—but see n.55 below.

[49] *Rhyth.* 2.20 = p.22.19–29 Pighi, in particular 22.26–29 (see also the interesting explanation on ῥητόν and ἄλογον in rhythms at *Rhyth.* 2.21): ὁ γὰρ τοιοῦτοςποὺς ἄλογον μὲν ἕξει τὸ ἄνω πρὸς τὸ κάτω· ἔσται δ' ἡ ἀλογία μεταξὺ δύο λόγων γνωρίμων τῇ αἰσθήσει, τοῦ τε ἴσου καὶ τοῦ διπλασίου. καλεῖται δ' οὗτος χορεῖος ἄλογος. This *choreios* foot is generated when the long of a dactylic foot is shorter than the perfect long; the same phenomenon occurring in the anapest gives rise to the "cyclic" foot: Dion. Hal. *Comp*. 17.12 (123.12–17 Aujac-Lebel, with discussion at 21–25 and references in the "Note complémentaire" at 212). For ἄλογα διαστήματα in harmonic theory see [Plut.] *De mus*. 39, 1145D.

[50] *Comp*. 20.21 (145.14 Aujac-Lebel, with references at 217).

sound that retains one and the same tone. Hence each taken alone is ἄλογος, for it is one and undifferentiated in relation to itself, whereas ratio is a relation and occurs first in two terms." The basic entities of harmonic theory are the intervals, that is, the relations between pairs of notes; notes taken in isolation are irrelevant to melody, as Ptolemy will explain in the subsequent sentence: "in a comparison between two notes, when they are unequal-toned, it makes a ratio from the quantity by which one exceeds the other, and it is in these that the melodic and the unmelodic appear."[51]

Finally, one also finds a decidedly metaphorical use of ἄλογος, still in a scientific domain: Herophilus' theory of human pulse, modelled on rhythmic theory.[52] Herophilus defines the rhythm associated with pulse as the ratio between the time of dilation and the time of contraction, and holds that any of these times, at an assigned age of human life, is an integer multiple of the time of dilation or of contraction of the newborn child (in whom these times are equal). Herophilus then sets up a rhythmic model of "normal" pulse-rhythms: a short is assigned to the primary time-unit, a long to any time of dilation or of contraction longer than this. In this way, the pulse-rhythm is represented by a metrical foot: the basic rhythm of the newborn child is represented by the pyrrhic foot (short dilation, short contraction), that of the growing child by the trochee (long, short), that of full-grown man by the spondee (long, long),[53] old people having a iambic pulse-rhythm (short,

[51] *Harm.* 1.4 (10.19–23 Düring; transl. Barker): φθόγγος ἐστὶ ψόφος ἕνα καὶ τὸν αὐτὸν ἐπέχων τόνον. διὸ καὶ μόνος μὲν ἕκαστος ἄλογος, εἷς γὰρ καὶ πρὸς ἑαυτὸν ἀδιάφορος, ὁ δὲ λόγος τῶν πρός τι καὶ ἐν δυσὶ τοῖς πρώτοις. κατὰ δὲ τὴν πρὸς ἀλλήλους, ὅταν ὦσιν ἀνισότονοι, παραβολὴν ποιεῖ τινα λόγον ἐκ τοῦ ποσοῦ τῆς ὑπεροχῆς, ἐν οἷς δὴ τό τε ἐκμελὲς ἤδη καταφαίνεται καὶ τὸ ἐμμελές. See also the explanation given by Porphyry at *In Harm.* 87.25–88.16 Düring.

[52] See H. von Staden, *Herophilus. The Art of Medicine in Early Alexandria* (Cambridge 1989) 276–284 and frr.172–185.

[53] Taking up another mathematical term, Herophilus calls this pulse-rhythm διὰ ἴσου (a standard manipulation of ratios: see Euc. *Elem*. 5.def.17

long). Herophilus held that the pulse of the newborn child

> is constituted ἄλογον. He calls the pulse which does not bear a proportion with respect to some "pulse" an ἄλογον pulse, for it has neither a double ratio, nor a ratio of one and a half to one, nor any other ratio, but rather is completely short, and we observe it to be similar in size to the prick of a needle. For this reason Herophilus called it ἄλογον, as one should.[54]

The reason for the newborn child's pulse being ἄλογος lies in the fact that the ordered pair time-of-dilation/time-of-contraction is the unit defining the pulse-system, and this unit cannot have a ratio to itself. In the same way, to writers like Nicomachus or Iamblichus, the ratio of equality (in particular if it is conceived as the ratio of one to one) is of a different nature than the other ratios, its function being more properly that of a principle for the more complex system of relations of inequality.[55]

We see, thus, that another way of being ἀ-λόγος in the arithmetical domain is simply to be the "unit" of a particular system that admits of a numerical model, since this cannot bear any

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and proposition 5.22).

[54] Rufus *Syn.puls.* 4.3 (= fr.177 von Staden; transl. von Staden, with modifications): τοῦτον τὸν σφυγμὸν Ἡρόφιλος <u>ἄλογον</u> συνεστάναι φησίν· <u>ἄλογον</u> δὲ καλεῖ σφυγμὸν τὸν μὴ ἔχοντα πρός τινα ἀναλογίαν· οὔτε γὰρ τὸν διπλάσιον, οὔτε τὸν ἡμιόλιον, οὔτε ἕτερόν τινα <u>λόγον</u> ἔχει οὗτος, ἀλλά ἐστι βραχὺς παντελῶς καὶ τῷ μεγέθει βελόνης κεντήματι ὁμοίως ἡμῖν ὑποπίπτει· διὸ καὶ πρῶτον αὐτὸν Ἡρόφιλος <u>ἄλογον</u> δεόντως εἶπεν.

[55] See Nic. *Ar.* 1.17.4 and Iambl. *In Nic.* 3.37–38 (112.24–32 Vinel = 43.22–44.7 Pistelli). In this context, one should not forget the formidable Platonic wordplay (whose mathematical connotations are obvious given Theaetetus' achievement on classifying "powers" celebrated at *Tht.* 147C–148B) about στοιχεῖα <u>ἄλογα</u> καὶ ἄγνωστα as opposed to συλλαβὰς γνωστάς τε καὶ <u>ῥητάς</u> (*Tht.* 202B6–7) underlying the argument developed at 202B–204A: again, the basic elements of a complex system such as speech are quite aptly termed ἄλογα—"unaccountable," in Levett's translation. It is a general feature of Greek thought, most notably in mathematical contexts, to regard the principles of a system of entities as having a different nature than that of the elements of the generated system, but here the point is to call such principles ἄλογα.

relations with (that is, ratios to) itself. It would have been interesting to see Diophantus striving to invent a name for the μονάς as a 0-species, and Anatolius/Psellus to find an ordinal to attach to this very peculiar ἄλογος.

5. *Complement: the sign for the ἀριθμός*

As for the sign for the ἀριθμός, Tannery prints an inverted *stigma*; Heath has it as a final *sigma*, as I have done above.[56] The signs featuring in the *Matritensis* and, to a lesser extent, in the other manuscripts also are S-shaped. The problem is that this sign coincides both with one of the most current abbreviations of ἀριθμός[57] and with one of the most current abbreviations of καί (not to mention the fact that it also represents the numeral "six"). However, as all manuscripts consistently have, a graphic tool was at hand in order to differentiate between discursive objects (abbreviations) and metadiscursive objects (signs): overlining the signs (that is, putting a macron on them), a tool used for instance to mark numeral letters and the denotative letters occurring in geometric proofs.[58] As for differentiating the ab-

[56] On the sign for the ἀριθμός in the *Arithmetica* see Tannery's remarks at *Diophanti opera* II XL–XLI; Heath, *Diophantus* 32–37. Neither author was in a position to take into account the evidence of *P.Mich.* III 144, on which see below.

[57] Note the difference: basically the same graphic entity (the grapheme here represented by ς) is at the same time an *abbreviation* of the part of speech ἀριθμός, used in its current meaning within the Diophantine sentence (discursive function) and a *sign* of the arbitrary designation ἀριθμός mentioned in the same sentence (metadiscursive function). For the problems raised by the interplay between signs and abbreviations when syntagms designating mathematical entities are at issue, see F. Acerbi, "Funzioni e modalità di trasmissione delle notazioni numeriche nella trattatistica matematica greca: due esempi paradigmatici," *S&T* 11 (2013) 123–165.

[58] The overhanging bar identifies a string of signs that does not have a proper grammatical or syntactical function in the discourse. This happens in particular when the string of signs does not give rise to a Greek word: these are denotative and numeral letters, as said, but also terms originating in other idioms, or contractions of Greek words like the *nomina sacra* (see in the first place L. Traube, *Nomina sacra* [Munich 1907] 45–47); such terms can possibly be preceded by a "citational" neuter article, depending on

breviations of ἀριθμός and of καί, this was the function of the compendia for terminations (in our case, a supralinear *omicron*) and related accentuation marks.[59] The result is what we read in the *Matritensis*: the first ϛ (abbreviation) carries a supralinear *omicron* and a grave accent,[60] the second ϛ (abbreviation) only has a grave accent, the third ϛ (sign) a macron. As a matter of fact, the copyist did not do a perfect job,[61] for he unduly added a grave accent, just after the macron, to the third ϛ (a sign cannot have an accent); what is more, he always expanded elsewhere the sign to suitable forms of ἀριθμός.

As we have seen in the apparatus to the Diophantine sentence in the *Matritensis*, John Chortasmenos (†1431), the author

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what use is made of them in the argument. But this is not the only possibility: the grammatical papyri and the earliest manuscripts of Greek grammatical treatises mark by means of macrons the examples of the parts of speech at issue (and these are "true" Greek words): so *Par.gr.* 2548, *codex vetustissimus* and the only witness of the "minor works" of Apollonius Dyscolus; f. 106ᵛ, where the macrons are conspicuous, is reproduced as plate 19 in G. De Gregorio, "Materiali vecchi e nuovi per uno studio della minuscola greca fra VII e IX secolo," in G. Prato (ed.), *I manoscritti greci tra riflessione e dibattito* (Florence 2000) 83–151 (137–138 for the date of the manuscript). For the papyri see A. Wouters, *The Grammatical Papyri from Graeco-Roman Egypt* (Brussels 1979), *passim*; these papyri usually comprise lists of parts of speech, none of which is preceded by the "citational" article. In all these cases, the macron is the graphic counterpart of the distinction between *mention* and *use* (as a part of speech in the ongoing discourse) of a string of characters, that is, between denotative and semantic function. The abbreviations do not require macrons since their graphic features—which include non-alphabetic elements such as compendia for terminations, letters *supra lineam*, marks of contraction or of suspension such as slashes or bars possibly singling out only a subset of the string of alphabetic signs—automatically exclude them from the set of "possible terms of the Greek language."

[59] But confusion between these two terms on the basis of a misinterpreted abbreviation is one of the most widespread errors encountered in mathematical manuscripts. For Diophantus see Tannery at *Diophanti opera* II XXXV.

[60] The two Vatican manuscripts also add the breathing.

[61] This is the first copyist of the *Matritensis*, whose work on the *Arithmetica* ends at the fourth-to-last line of f. 62ʳ.

of extensive annotations *infra lineam* to the *Arithmetica* in this codex,[62] glosses the first ς with ἀριθμός and the third with the indication καὶ ἔστιν αὐτοῦ σημεῖον τόδε, followed by a sign widely used by later Byzantine copyists and almost identical with the canonical abbreviation of οὖν. It goes without saying that there is no guarantee that the sign originally introduced by Diophantus for the ἀριθμός also coincided with an abbreviation of ἀριθμός. On the other hand, the evidence of *P.Mich.* III 144 (2nd cent. init.), the only such piece of evidence in which this sign appears, strongly suggests that the *Matritensis*, and the entire medieval tradition of the *Arithmetica* with it, faithfully reproduces, while accentuating its sinuosity, the original sign.[63]

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[62] See Pérez Martín, *Maxime Planude* 450; F. Acerbi, "Why John Chortasmenos sent Diophantus to the Devil," *GRBS* 53 (2013) 379–389.

[63] The papyrus is edited in C. E. Robbins, "P. Mich. 620: A Series of Arithmetical Problems," *CP* 24 (1929) 321–329; a reproduction can be found at http://quod.lib.umich.edu/cgi/i/image/image-idx?c=apis&page=search, inventory number 620. The S-shaped transcription in Robbins' article is quite faithful to the form the sign has in the papyrus.