Ones

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DIOPHANTUS' *ARITHMETICA* is a collection of arithmetical[1] problems: to find numbers satisfying specific conditions that are stated in the enunciation.[2] For instance, *Ar.* 1.27 requires to find two numbers such that their sum and their product are assigned numbers. Each problem of the *Arithmetica* is solved by concretely setting out the assigned numbers (in the case of 1.27, they are given as 20 and 96), by positing one unknown and by solving the equality ('equation' in our language) resulting from the constraints stipulated in the enunciation. Apart from this very general approach, there is no standard method to solve any specific set of Diophantine problems: the *Arithmetica* presents a host of clever tricks and specific manipulations, which a student gets acquainted with by simply doing Diophantine problems.

Problem 5.30 begins with an epigram:[3]

[1] 'Arithmetic' is the ancient denomination of our 'number theory'. The discipline explaining how to calculate with specific numbers was called 'logistic'.

[2] The Diophantine writings were edited by P. Tannery, *Diophanti Alexandrini opera omnia cum graecis commentariis* I–II (Leipzig 1893 text and transl., 1895 *pseudepigrapha*, *testimonia*, *scholia*, *index graecitatis*). A new edition of the *Arithmetica* has been provided in A. Allard, *Diophante d'Alexandrie, Les Arithmétiques* I–II (diss. Louvain 1980); it lies unpublished and, even in the University Library of Louvain-la-Neuve, only the volume containing text and translation is available. As we shall see, Allard's edition is no improvement on Tannery's. The *Arithmetica* was paraphrased in English and commented on extensively in T. L. Heath, *Diophantus of Alexandria. A Study in the History of Greek Algebra* (Cambridge 1910).

[3] I 384.6–21 Tannery; in the manuscripts, the numeral letters are identified by macrons. The epigram, whose text needs emendation, was studied

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ὀκταδράχμους καὶ πενταδράχμους χοέας τις ἔμιξε
τοῖς ὁμοπλοῖσι ποιεῖν χρήστ' ἐπιταττόμενος,
καὶ τιμὴν ἀπέδωκεν ὑπὲρ πάντων τετράγωνον,
τὰς ἐπιταχθείσας δεξάμενον μονάδας
καὶ ποιοῦντα πάλιν ἕτερόν σε φέρειν τετράγωνον
κτησάμενον πλευρὰν σύνθεμα τῶν χοέων·
ὥστε διάστειλον τοὺς ὀκταδράχμους πόσοι ἦσαν,
καὶ πάλι τοὺς ἑτέρους, παῖ, λέγε πενταδράχμους.

The meaning of the epigram is explained as follows:

ἠγόρασέν τις <u>β′ ἑνῆ οἴνου, ἐκ μὲν τοῦ ἑνὸς τὸν χοέα δραχμῶν η′, ἐκ δὲ τοῦ ἑνὸς τὸν χοέα δραχμῶν ε</u>, καὶ ἀπέδωκεν ὑπὲρ πάντων τιμὴν τετράγωνον ἀριθμόν, ὃς πρὸς μ^{o} ξ′ ἐποίει τετράγωνον πλευρὰν ἔχοντα τὸ πλῆθος τῶν χοέων· διάστειλον τοὺς ὀκταδράχμους καὶ πενταδράχμους.

Someone bought <u>2 ἑνῆ of wine—a congius of the one being worth 8 drachms, a congius of the other being worth 5 drachms</u>—and paid for all of them a square number, which, added to 60 units, made a square having the multiplicity of congii as its side: tell apart the eight-drachms and the five-drachms.[4]

The underlined pericope contains, untranslated, the term I shall focus on: ἑνῆ, apparently a *hapax legomenon* in the whole ancient Greek *corpus*, including papyri and inscriptions. The passage was paid only marginal attention in the available critical editions of the Diophantine writings, and has never been discussed in the literature. The aim of the present note is to offer such a discussion. Before addressing grammatical problems, however, it is necessary to briefly present the *Arithmetica* and say something about its text.

The *Arithmetica* was originally redacted in thirteen books.[5] A

in P. Tannery, "Sur une épigramme attribuée à Diophante," *REG* 4 (1891) 377–382, repr. *Mémoires scientifiques* II (Toulouse/Paris 1912) 433–439.

[4] An algebraic transcription of this enunciation would read: $mx + ny + a = (x + y)^2$, with $mx + ny$ being a square number. In our text, x and y are the sought numbers, and the parameters m, n, a are the assigned values 8, 5, 60, respectively.

[5] So Diophantus himself in his introduction: I 16.6–7.

large collection of problems arranged in six books is preserved in Greek.[6] It is not clear whether such a division exactly reflects the original Diophantine partition, or rearrangements of the material have occurred at some stages of the transmission. In fact, the criterion by which the material of the Greek *Arithmetica* is organized into books is not always obvious, if indeed any general criterion is at work.[7] As a consequence of this, not every manuscript of the Greek *Arithmetica* has it in six books; three of them divide Book 4 into two books, at least two other manuscripts divide Book 1 into two books.[8] Even if the manuscripts in question are late apographs, it is by no means assured that a similar phenomenon could not have occurred in very early copies. The loose deductive structure of the *Arithmetica* has also the consequence of easily allowing unauthentic material to creep into the text: Tannery deemed problems 2.1–7 and 3.1–4 spurious, on the grounds that they contain material more properly pertaining to the final parts of Books 1 and 2, respectively, or that they repeat other propositions. Tannery suggested that those problems were *marginalia*, coming from a commentary on Books 1 and 2, that got inserted into the text.[9] After comparison with the Arabic text, where problems 6.1–11 are likely interpolations, problems 4.1–2 of the Greek text

[6] Four books of problems are transmitted in Arabic translation, referred to in the titles and the subscriptions as Books 4 to 7 of Diophantus' treatise. The sets of Greek and of Arabic problems are almost completely non-overlapping.

[7] Local criteria may apply; for instance, the sixth book contains problems of quite a distinctive kind.

[8] See the description of the manuscripts in A. Allard, "La tradition du texte grec des *Arithmétiques* de Diophante d'Alexandrie," *RHT* 12–13 (1982–1983) 57–137, at 58–72.

[9] P. Tannery, "La perte de sept livres de Diophante," *Bulletin des Sciences mathématiques* 4 (1884) 192–206, repr. *Mémoires scientifiques* II 73–90, at 80–82, and *Diophanti opera* I 83 and 139 *in app*. Tannery suspected also problems 2.17–18 and 3.20–21, since they are identical with 1.22–23 and 2.15 and 14, respectively.

should also be regarded as spurious.[10]

There are at least two reasons to suspect the authenticity of *Ar.* 5.30.[11] First, even if an epigram was a way of enunciating arithmetical problems or riddles that is well-established in the ancient *corpus*,[12] 5.30 is the only problem of the *Arithmetica* that is formulated in this way. Second, 5.30 is in fact the last problem of Book 5, and we have seen that some problems near the end of other books have been considered later additions by the editors. On the other hand, the authenticity of 5.30 seems confirmed by a series of facts: the problem is a truly difficult one, its contents are homogeneous with those of the problems preceding it, the style in which the proof is redacted is definitely Diophantine. To reconcile this contradictory evidence, we might entertain the hypothesis that a late revisor modified the beginning and the end of the original problem in order to set it in its present form:[13] it is an easy exercise to do it—to be fair, the same exercise could successfully be applied to any problem

[10] See J. Sesiano, *Books IV to VII of Diophantus'* Arithmetica, *in the Arabic Translation Attributed to Qustā ibn Lūqā* (New York 1982) 53.

[11] Allard, *Diophante* 935, deems the problem spurious but I could not have access to the volume of his thesis where he discusses the issue.

[12] Epigrams 14.116–146 of the *Anthologia Palatina* are arithmetical problems; 14.126 deals exactly with the life-span of Diophantus. On the structure of this collection of arithmetical epigrams see P. Tannery, "Sur les épigrammes arithmétiques de l'Anthologie palatine," *REG* 7 (1894) 59–62, repr. *Mémoires scientifiques* II 442–446.

[13] The bulk of the problem (I 386.5–390.2) bears no trace of the (fictionally) practical context of the enunciation. On the other hand, the epigram and the explanatory enunciation contain the only two occurrences of the verb διαστέλλω "to tell apart" in the whole *Arithmetica*. This is a technical term of the metrological domain and it is not used appropriately in our passage: it usually features in 'problems of separation', where what is added and has to be "told apart" are unhomogeneous magnitudes like line segments and surfaces. We find a number of such problems in the pseudo-Heronian *Geometrica*, in fact a philological patchwork of disparate collections of metrological problems, assembled by their editor J. L. Heiberg and printed in *Heronis opera omnia* IV: see *Geom.* 21.9–10, 24.3, 24.10–13, 24.43, 24.46–47.

of the *Arithmetica*. Be that as it may: even if 5.30 was authored by some later scholiast and not by Diophantus himself (who probably lived about 150 CE and surely writes in a late Greek), we may safely submit that the *hapax* ἑνῆ was written on purpose by some Greek native speaker. But was it?

The *apparatus criticus* of Tannery's edition is notoriously unreliable, and one must check his readings against the manuscripts. Now, the rich tradition of the *Arithmetica* (31 witnesses) can readily be reduced to no more than four independent manuscripts: *Matrit.* 4678, *Vat.gr.* 191 and 304, and *Marc.gr.* 308, the latter in fact containing a recension made by the renowned scholar Maximus Planudes (†1305).[14] The readings are *Matrit.* 4678, f. 120^{r} ἑνῆ; *Vat.gr.* 191, f. 386^{v} ἑνῆ; *Vat.gr.* 304, f. 112^{r} ἐνῆ; *Marc.gr.* 308, f. 244^{r} ἐνῆ: the only variant is in the breathing. Tannery, who collated only the first and the last manuscripts, correctly puts ἑνῆ in the main text and relegates the *vox nihili* ἐνῆ to the apparatus.[15] I say "correctly" since the immediately subsequent correlative ἐκ μὲν τοῦ ἑνὸς … ἐκ δὲ τοῦ ἑνὸς … leaves no doubt as to the meaning of ἑνῆ: it is a plurality of 'ones'.

The interesting point is how this plural was formed. The only scholar who has addressed this problem is Tannery, who translated the beginning of the clause containing ἑνῆ "Quidam vinum emit duarum qualitatum," and who, in his *index graecitatis*, recorded the term s.v. ἑνῆ, adding "(ut plurale vocis ἕν?)."[16] This suggestion is wrong, as we shall see by turning our attention to the attested plural forms of ἕν.

[14] The most recent analysis of the manuscript tradition of the *Arithmetica* was provided by Allard, *RHT* 12–13 (1982–1983) 57–137, who unconvincingly argues that the first three manuscripts are independent copies of a lost archetype. A closer look, in fact, shows that the whole non-Planudean tradition depends on the *Matritensis*.

[15] To be fair, also ἑνῆ with rough breathing was a *vox nihili* until now.

[16] I 385 and II 270, respectively. Heath, *Diophantus* 224, translates "a certain number" and does not add any comment. Allard, *Diophante* 933, translates "deux sortes." His apparatus at 896 does not record any variants(!) for the ἑνῆ he writes in his text.

How to inflect 'one' in the plural: a survey of the relevant texts

Admittedly, forming the plural of an adjective whose meaning is 'one' looks like doing something self-contradictory, and we shall see that some ancient grammarians agreed with this *prima facie* impression. Yet, there are contexts in which dealing with pluralities of 'ones' is perfectly natural, and this fact makes in turn necessary to create adequate lexical tools.

The context most conducive to introducing a plurality of 'ones' is obviously the science of arithmetic, especially in view of the fact that numbers were conceived as collections of units (see below). Since nothing has survived of pre-Euclidean arithmetic, the first author where we read 'one' in the plural is Aristotle. This happens in three passages; in two of them the immediate context is strictly arithmetical (a number is "many ones"),[17] even if they are embedded in more philosophically-coloured discussions about the potential infinity and about the dialectic between one and many, respectively. A third passage introduces a polemical, and more markedly philosophical, dimension: the targets are unidentified Platonizing philosophers, who claim that numbers have a primary ontological status, and that the 'one' is their principle—yet, Aristotle replies, it is absurd that there is a 'one' which is first of 'ones',

[17] *Ph.* 4.7, 207b5–10: αἴτιον δ' ὅτι τὸ ἕν ἐστιν ἀδιαίρετον, ὅ τι περ ἂν ἓν ᾖ (οἷον ἄνθρωπος εἷς ἄνθρωπος καὶ οὐ πολλοί), ὁ δ' ἀριθμός ἐστιν ἕνα πλείω καὶ πόσ' ἄττα, ὥστ' ἀνάγκη στῆναι ἐπὶ τὸ ἀδιαίρετον (τὸ γὰρ τρία καὶ δύο παρώνυμα ὀνόματά ἐστιν, ὁμοίως δὲ καὶ τῶν ἄλλων ἀριθμῶν ἕκαστος), and *Metaph.* I 6, 1056b20–25: οὕτως γὰρ λέγομεν ἓν ἢ πολλά, ὥσπερ εἴ τις εἴποι ἓν καὶ ἕνα ἢ λευκὸν καὶ λευκά, καὶ τὰ μεμετρημένα πρὸς τὸ μέτρον [καὶ τὸ μετρητόν]· οὕτως καὶ τὰ πολλαπλάσια λέγεται· πολλὰ γὰρ ἕκαστος ὁ ἀριθμὸς ὅτι ἕνα καὶ ὅτι μετρητὸς ἑνὶ ἕκαστος, καὶ ὡς τὸ ἀντικείμενον τῷ ἑνί, οὐ τῷ ὀλίγῳ. Here and henceforth, the relevant syntagmas are underlined. In the first passage, ὄνομα means "nominative singular" (*Cat.* 1, 1a12–15). Since τρία and δύο are necessarily in the nominative plural and nominative dual, respectively, when they are used as a nominative singular this can be only done by paronymy (*Cat.* 8, 10a27–b11): see M. Ugaglia, *Aristotele*, *Fisica. Libro III* (Rome 2012) 170.

as they say, but not a dyad which is first of dyads, and so on.[18] In this text, Aristotle does not resort to the same lexical material to denote the 'one' on the one hand and the numbers greater than it on the other: he writes τὸ ἓν μὲν εἶναί τι πρῶτον τῶν ἑνῶν […] δυάδα δὲ τῶν δυάδων μή, μηδὲ τριάδα τῶν τριάδων: 'one' is the substantivized numeral adjective τὸ ἕν, whereas δυάς and τριάς are numeral substantives. Aristotle might well have written[19] ἡ ἑνὰς μὲν εἶναί τι πρῶτον τῶν ἑνάδων or, better still, ἡ μονὰς μὲν εἶναί τι πρῶτον τῶν μονάδων, the noun μονάς being repeatedly used by Aristotle himself in the arguments immediately preceding this very passage. Quite likely, then, the nexus τὸ ἓν … τῶν ἑνῶν is intended to strengthen the connection with the criticized source.

If ἕν and μονάς were used interchangeably by Aristotle[20] and, as we shall see, in the early philosophical tradition, mathematical exigencies induced a differentiation of their semantic range, a move that in its turn triggered further philosophical speculations about the difference between the two terms.

Before seeing this, one must recall that numeral substantives, not substantivized numeral adjectives, were normally used to designate numbers as abstract entities: ἡ δυάς instead of τὰ δύο, ἡ τριάς instead of τὰ τρία, etc.[21] This happens most

[18] *Metaph*. M 8, 1083a21–27: εἰσὶ δ' οὗτοι ὅσοι ἰδέας μὲν οὐκ οἴονται εἶναι οὔτε ἁπλῶς οὔτε ὡς ἀριθμούς τινας οὔσας, τὰ δὲ μαθηματικὰ εἶναι καὶ τοὺς ἀριθμοὺς πρώτους τῶν ὄντων, καὶ ἀρχὴν αὐτῶν εἶναι αὐτὸ τὸ ἕν. ἄτοπον γὰρ τὸ ἓν μὲν εἶναί τι πρῶτον τῶν ἑνῶν, ὥσπερ ἐκεῖνοί φασι, δυάδα δὲ τῶν δυάδων μή, μηδὲ τριάδα τῶν τριάδων· τοῦ γὰρ αὐτοῦ λόγου πάντα ἐστίν.

[19] The noun ἑνάς (in the plural form ἑνάδων) was firmly attested in the philosophical lexicon thanks to Plato *Phlb*. 15A4–7, where it refers to a list of four items, each qualified by a suitable form of the numeral adjective εἷς: ὅταν δέ τις ἕνα ἄνθρωπον ἐπιχειρῇ τίθεσθαι καὶ βοῦν ἕνα καὶ τὸ καλὸν ἓν καὶ τὸ ἀγαθὸν ἕν, περὶ τούτων τῶν ἑνάδων καὶ τῶν τοιούτων ἡ πολλὴ σπουδὴ μετὰ διαιρέσεως ἀμφισβήτησις γίγνεται.

[20] See e.g. the discussion of the mathematical number at *Metaph*. M 6–7.

[21] On all these forms see R. Kühner and F. Blass, *Ausführliche Grammatik der griechischen Sprache* I.1[3] (Hannover 1890) 621–624, §181.

naturally in number-theoretical contexts, and we read texts according to which a criterion that sets the theoretical science of arithmetic apart from lowly logistic is the use of the numeral substantives instead of the substantivized numeral adjectives.[22] If, as is only natural, we find both in strictly arithmetical texts, their ranges of application were strictly non-overlapping. It suffices to look at the enunciation of Euclid *Elem.* 9.8:[23]

> ἐὰν ἀπὸ μονάδος ὁποσοιοῦν ἀριθμοὶ ἐξῆς ἀνάλογον ὦσιν, ὁ μὲν τρίτος ἀπὸ τῆς μονάδος τετράγωνος ἔσται καὶ οἱ ἕνα διαλείποντες, ὁ δὲ τέταρτος κύβος καὶ οἱ δύο διαλείποντες πάντες, ὁ δὲ ἕβδομος κύβος ἅμα καὶ τετράγωνος καὶ οἱ πέντε διαλείποντες.
>
> If as many numbers as we please beginning from a unit be in continued proportion, the third from the unit will be square, as will also those which leave out one; the fourth will be cube, as will also all those which leave out two; the seventh will be at once cube and square, as will also those which leave out five.

In this sentence, μονάς is a number-theoretic entity and is hence designed by a numeral substantive, while the numeral adjectives count the tokens of a certain object (here the ἀριθμοί left out of the series). An enunciation where a numeral substantive is the direct object of the verb λείπειν as in *Elem.* 9.8 is that of Diophantus *Ar.* 5.6 (I 322.2–5):

[22] See the extract from Geminus in *Def.* 135.5, at *Heronis opera* IV 98.13–18: λογιστική ἐστι θεωρία ἡ τῶν ἀριθμητῶν, οὐχὶ δὲ τῶν ἀριθμῶν, μεταχειριστική, οὐ τὸν ὄντως ἀριθμὸν λαμβάνουσα, ὑποτιθεμένη δὲ τὸ μὲν ἓν ὡς μονάδα, τὸ δὲ ἀριθμητὸν ὡς ἀριθμόν, οἷον τὰ τρία τριάδα εἶναι καὶ τὰ δέκα δεκάδα, ἐφ' ὧν ἐπάγει τὰ κατὰ ἀριθμητικὴν θεωρήματα = beginning of scholium to *Chrm.* 165E6, in D. Cufalo (ed.), *Scholia Graeca in Platonem* I (Rome 2007) 173, no. 27. As for late Neoplatonism, see the whole discussion about subordination in sciences at Proclus *In Euc.* 38.1–41.2 Friedlein, in particular 40.2–9 (the entire argument relies heavily on Geminus).

[23] *Euclidis Elementa* II 194.5–9 Heiberg-Stamatis. Note also the presence of ordinal numerals and the absence of the article in the first occurrence of the syntagma ἀπὸ (τῆς) μονάδος; Greek arithmetic avails itself of an unlimited supply of μονάδες: in its first occurrence in each proposition, "a" unit is taken.

εὑρεῖν τρεῖς ἀριθμοὺς ὅπως ἕκαστος μὲν αὐτῶν λείψας δυάδα ποιῇ τετράγωνον, ὁ δὲ ὑπὸ δύο ὁποιωνοῦν, ἐάν τε λείψῃ συναμφότερον, ἐάν τε τὸν λοιπόν, ποιῇ τετράγωνον.

To find three numbers such that each of them minus a dyad[24] makes a square, and the ‹rectangle contained› by two as we please, either minus their sum or minus their difference, makes a square.

When numbers themselves are counted, the number designating the plurality is a numeral substantive, while the number giving the amount is a numeral adjective.[25] In more relaxed number-theoretic contexts (as in the extract from Domninus quoted in n.25), and invariably in logistic contexts, the names of numbers are expressed by numeral letters, preceded either by a substantivizing τά or by a ὁ, which reminds us that we are still speaking about an ἀριθμός. Occasionally, as e.g. in the Heronian *Metrica*, the numeral letters are preceded by a plural feminine article, intimating that one is really counting μονάδες.[26]

This brings us back to the mathematical lexicon. Defining 'number' as a multiplicity composed of units[27] requires one to define what a unit is. *Elem.* 7.def.1 does exactly this job: μονάς ἐστιν καθ' ἣν ἕκαστον τῶν ὄντων ἓν λέγεται.[28] This Euclidean

[24] The numeral substantive δυάς occurs 32 times in the *Arithmetica* and 8 in the *Elements*, prop. 9.32 (*ter*), 34 (*quater*), 36.

[25] E.g. Domninus *Ench.* 8, 108.24–26 Riedlberger: τῶν δὲ περιττῶν οἱ μὲν τέμνονται εἰς ἴσους τινὰς ἀριθμούς, ὡς ὁ θ′ εἰς τρεῖς τριάδας, ὡς ‹ὁ› ιε′ εἵς τε τρεῖς πεντάδας καὶ εἰς πέντε τριάδας, ὡς ὁ λε′ εἰς ε′ ἑβδομάδας καὶ εἰς ἑπτὰ πεντάδας.

[26] Hero made his choice on purpose, as he himself declares at *Metr.* 1 praef. (*Heronis opera* III 6.4–7).

[27] At *Elem.* 7.def.2: ἀριθμὸς δὲ τὸ ἐκ μονάδων συγκείμενον πλῆθος. The two definitions are at *Euclidis Elementa* II 103.2–4.

[28] Among the several definitions attested in ancient writings, Chrysippus' deserves a mention (Iambl. *In Nic.* 11.8–9 Pistelli = 2.10, 76.19 Vinel). It reads μονάς ἐστι πλῆθος ἕν, a paradoxical statement where envisaging a "unit multiplicity" is a semantic incongruity of the same kind as (and exactly the inverse of) Aristotle's resorting to the plural of 'one' in his description of

definition makes clear the difference between the noun-token and the predicate (ἓν λέγεται), gets rid of the grammatical problem of pluralizing the adjective 'one', and, finally and most importantly, by resorting to the stem μον- avoids the definitional vicious circle that would have arisen from presenting the stem ἑν- (as in ἕν itself or in ἑνάς) both in the *definiendum* and in the *definiens*. Such a lexical convention is adhered to without exceptions in the whole of the *Elements* and in all the subsequent number-theoretical tradition. This fact neutralizes the possibility that what we read as *Elem.* 7.def.1 can be dismissed as irrelevant in that this definition might well have been formulated, and put at the very beginning of the arithmetic books of the *Elements*, by forcing a preexisting definition to fit the strict denotative practice of the treatise.[29] It may also be that the philosophical connotations with which the term ἑνάς was charged already in early times (not to speak of τὸ ἕν, "the One" with which every Greek philosopher after Parmenides had to come to grips) might have suggested turning one's attention to μονάς.[30] The latter term almost surely belonged in the notional

ἀριθμός as ἕνα πλείω.

[29] A circular characterization of this kind was around well before Euclid, if we are to believe Sextus Empiricus *Math.* 4.11: τὴν τοῦ ἑνὸς τοίνυν νόησιν διατυπῶν ἡμῖν πυθαγορικώτερον ὁ Πλάτων φησὶν "ἕν ἐστιν οὗ μηδὲν χωρὶς λέγεται ἕν", the syntagma οὗ μηδὲν χωρίς being exactly the double negation of the Euclidean καθ' ἣν ἕκαστον τῶν ὄντων. Note however Sextus' formulation τὴν τοῦ ἑνὸς νόησιν διατυπῶν "outlining the notion of the one," which does not seem to entail a technical, definitional context: the aim of the reported characterization is only to establish a link between the Basic Entity (noun ἕν) and our categorizations (predicate ἕν)—it remains that one would have liked to read a τὸ before the first ἕν, and that the whole clause is dangerously near to a wordplay. Doubts about the authenticity of *Elem.* 7.def.1 are *a priori* legitimate, since this definition is totally ineffective from the mathematical point of view and (as a consequence) is never applied in the *Elements*.

[30] Recall the occurrence at Plato *Phlb.* 15A6 (n.19 above) and the fragment (*apud* Alexander *apud* Simplicius *In Ph.* 99.12–16 Diels and *apud* Philoponus *In Ph.* 42.9–17 Vitelli [Philoponus, however, does not mention any source]), where Eudemus lends Zeno the term ἑνάς for a description of

and terminological heritage of the early Pythagoreans, but in this case they did not show themselves very eager to fix a sharply defined lexicon.[31]

This rather plethoric terminological supply needed to be structured and hierarchized. The task was taken up by philosophically-oriented writers on technical matters, mainly of Middle- and Neoplatonic bent. A technical differentiation between ἕν and μονάς, amounting to that between logistic and arithmetic and putting due emphasis on the dichotomies 'intelligible'/'sensible' and 'indivisible'/'divisible', can be read in Theon of Smyrna.[32] Philosophically-marked speculations about the difference between ἕν and μονάς are attested in one of the pseudo-Heronian *Definitiones*, in a section that is a cento of extracts from Proclus and maybe other Neoplatonists. The author of this short text regards the ἕν as an entity which is "higher" than the μονάς, and in fact a principle of it and of the dyad. The text, however, suffers from a disappointing confu-

τὰ πολλὰ as πλῆθος ἑνάδων (Philoponus interchanges the two genera and writes τὸ πλῆθος ἐκ πλειόνων ἑνάδων σύγκειται).

[31] This much is expressly asserted by Theon of Smyrna, *Exp*. 20.19–20 Hiller: Ἀρχύτας δὲ καὶ Φιλόλαος ἀδιαφόρως τὸ ἓν καὶ μονάδα καλοῦσι καὶ τὴν μονάδα ἕν, and is implicit in the quotation in Iamblichus *In Nic*. 77.9–11 Pistelli (= 4.86, 146.7–8 Vinel), making up fr.44 B 8 D.-K. of Philolaus: ἡ μὲν μονὰς ὡς ἂν ἀρχὴ οὖσα πάντων κατὰ τὸν Φιλόλαον ("οὐ γὰρ ἕν" φησιν "ἀρχὰ πάντων"). But note that the doctrine assigning to the Unity the role of a first principle is quite definitely Neoplatonic, not early Pythagorean: for them, the numbers themselves have the status of principle and matter of everything, their elements being the even and the odd; the elements of the latter are in their turn the Bounded and the Unbounded (the correspondence is chiastic), while the 'one' (which is both even and odd) proceeds from both of these (Arist. *Metaph*. A 5, 986a15–21). On this fragment see C. A. Huffman, *Philolaus of Croton, Pythagorean and Presocratic* (Cambridge 1993) 345–346.

[32] *Exp*. 19.18–20.4: καὶ μονὰς τοίνυν ἐστὶν ἡ τοῦ ἑνὸς ἰδέα ἡ νοητή, ἥ ἐστιν ἄτομος· ἓν δὲ τὸ ἐν αἰσθητοῖς καθ' ἑαυτὸ λεγόμενον, οἷον εἷς ἵππος, εἷς ἄνθρωπος. ὥστ' εἴη ἂν ἀρχὴ τῶν μὲν ἀριθμῶν ἡ μονάς, τῶν δὲ ἀριθμητῶν τὸ ἕν· καὶ τὸ ἓν ὡς ἐν αἰσθητοῖς τέμνεσθαί φασιν εἰς ἄπειρον, οὐχ ὡς ἀριθμὸν οὐδὲ ὡς ἀρχὴν ἀριθμοῦ, ἀλλ' ὡς αἰσθητόν. ὥστε ἡ μὲν μονὰς νοητὴ οὖσα ἀδιαίρετος, τὸ δὲ ἓν ὡς αἰσθητὸν εἰς ἄπειρον τμητόν.

sion between ἕν and ἑνάς.[33] More refined distinctions between ἕν and μονάς are presented, just after the 'technical' one we have mentioned, again by Theon of Smyrna,[34] who explains among other things why Plato in the *Philebus* employed ἑνάς: it is a third species of 'one', namely, "a unit participating of the one" (μονὰς μετοχῇ τοῦ ἑνός).

Neo-Pythagorean authors such as Nicomachus (who does not even define the μονάς), Iamblichus, Anatolius, and in general all the arithmological literature, were apparently not interested in drawing such subtle distinctions, since their philosophical reference systems did not envisage hierarchies among objects as

[33] *Def.* 136.28, in *Heronis opera* IV 132.22–134.3: ἔστι διαφορὰ μονάδος καὶ ἑνάδος οὕτως· ἐπειδὴ ἔστιν ἐν τοῖς οὖσιν εἰδοποιία καὶ ταυτότης, καλεῖται μονάς. ἔστι δὲ ἑτερότης· καλεῖται δυάς. ἔστιν ἑτέρα ὑπερτέρα δύναμις, ἀρχὴ κοινὴ τῶν δύο τούτων, ἥτις πάντα ἐπίσταται· αὕτη ἓν καλεῖται. ὥστε τὸ ἓν ὑπέρτερόν ἐστι τῆς μονάδος. ἰστέον δέ, ὅτι, ἐπειδὴ ἔστι δυὰς καὶ μονὰς καὶ τὸ ἕν, δυὰς μὲν αὐτὰ τὰ σώματα, μονὰς δὲ τὸ εἶδος τὸ ἐν αὐτοῖς, ἓν δὲ ἡ φύσις. The passage (which very likely conflates two characterizations) has no parallels in other authors.

[34] *Exp.* 20.12–19: οἱ δὲ καὶ αὐτῶν τούτων ἀρχὴν τὴν μονάδα φασὶ καὶ τὸ ἓν πάσης ἀπηλλαγμένον διαφορᾶς ὡς ἐν ἀριθμοῖς, μόνον αὐτὸ ἕν, οὐ τὸ ἕν, τουτέστιν οὐ τόδε τὸ ποιὸν καὶ διαφοράν τινα πρὸς ἕτερον ἓν προσειληφός, ἀλλ' αὐτὸ καθ' αὑτὸ ἕν. οὕτω γὰρ ἂν ἀρχή τε καὶ μέτρον εἴη τῶν ὑφ' ἑαυτὸ ὄντων, καθὸ ἕκαστον τῶν ὄντων ἓν λέγεται, μετασχὸν τῆς πρώτης τοῦ ἑνὸς οὐσίας τε καὶ ἰδέας, which the sentence quoted above (n.31) immediately follows. Another point of view on the difference between ἕν and μονάς is related at *Exp.* 21.7–19: ἔνιοι δὲ ἑτέραν διαφορὰν τῆς μονάδος καὶ τοῦ ἑνὸς παρέδοσαν. τὸ μὲν γὰρ ἓν οὔτε κατ' οὐσίαν ἀλλοιοῦται, οὔτε τῇ μονάδι καὶ τοῖς περιττοῖς αἴτιόν ἐστι τοῦ μὴ ἀλλοιοῦσθαι κατ' οὐσίαν, οὔτε κατὰ ποιότητα (αὐτὸ γὰρ μονάς ἐστι καὶ οὐχ ὥσπερ αἱ μονάδες πολλαί) οὔτε κατὰ τὸ ποσόν (οὐδὲ γὰρ συντίθεται ὥσπερ αἱ μονάδες ἄλλῃ μονάδι)· ἓν γάρ ἐστι καὶ οὐ πολλά, διὸ καὶ ἑνικῶς καλεῖται ἕν. καὶ γὰρ εἰ παρὰ Πλάτωνι ἑνάδες εἴρηνται ἐν Φιλήβῳ, οὐ παρὰ τὸ ἓν ἐλέχθησαν, ἀλλὰ παρὰ τὴν ἑνάδα, ἥτις ἐστὶ μονὰς μετοχῇ τοῦ ἑνός. κατὰ πάντα δὴ ἀμετάβλητον τὸ ἓν τὸ ὡρισμένον τοῦτο ἐν τῇ μονάδι. ὥστε διαφέροι ἂν τὸ ἓν τῆς μονάδος, ὅτι τὸ μέν ἐστιν ὡρισμένον καὶ πέρας, αἱ δὲ μονάδες ἄπειροι καὶ ἀόριστοι. Note the adverb ἑνικῶς, which scores 554 occurrences in the ancient Greek *corpus*; the adjective ἑνικός denotes the grammatical "singular," and so does the adverb in this clause.

far as their ontological status is concerned. For this reason, and since the early Pythagorean tradition apparently did not pay attention to the term, the noun ἑνάς has zero occurrences in these writings.[35] On the other extreme of the spectrum, a vertiginous lexical proliferation took place in the writings of the late Neoplatonic philosophers. Their ontology included complex hierarchies of entities, each of which had its share of uniqueness and priority, let us say of 'oneness': it was "a system that, after the One-Good and at the summit of multiple levels of the intelligible, preserves a place for the 'henads'."[36] From this came the exigency of mobilising the widest possible terminological apparatus: ἕν, ἑνάς,[37] μονάς—and, what is important, of inflecting these terms in the plural. Even taking into account the bias due to the fact that Neoplatonic writings make up most of the philosophical record coming from antiquity,[38] it remains the case that a remarkable number of the TLG occurrences of such and related terms come from Neoplatonic authors. To give some figures, of the 8774 occurrences of the noun μονάς, 1011 are in Proclus, 467 in Damascius.[39] As is to be expected, the substantive ἑνάς has the best score among the

[35] The only exception, at *Theol.ar*. 76.14–15 De Falco, is a *calembour* between ἐννεάς and ἑνάς just at the beginning of the section on the ennead.

[36] Ph. Hoffmann, "What was Commentary in Late Antiquity? The Example of the Neoplatonic Commentators," in M. L. Gill and P. Pellegrin (eds.), *A Companion to Ancient Philosophy* (Malden 2006) 597–622, at 598.

[37] As seen above, there is a reason for the Neoplatonists being fond of this term: the occurrence in Plato *Phlb*. 15A6, already an object of exegesis in the Middle-platonic Theon of Smyrna, *Exp*. 21.14–16 (it is the underlined sentence in n.34). Note that Syrianus, *In Metaph*. 183.24 and 194.29 Kroll, had no problems in recognizing ἑνάς and μονάς as synonyms.

[38] See R. Goulet, "La conservation et la transmission des textes philosophiques grecs," in C. D'Ancona (ed.), *The Libraries of the Neoplatonists* (Leiden 2007) 29–61.

[39] The adjective μοναδικός scores 838 occurrences; only 10 of them are in Damascius, 175 in Proclus. Of the adjective μοναδιαῖος there are only 5, all in technical contexts: *Metr.* 2 praef. (*bis*: *Heronis opera* III 94.3.6) and *Prolegomena ad Almagestum* (*ter*: *Diophanti opera* II 7.19.23, 8.26).

Neoplatonists: of 854 occurrences, 398 of them in the plural(!), 420 are in Proclus, 164 in Damascius.[40] The fact that both in Proclus and in Damascius[41] we find plural forms of ἕν brings us back to the primary object of this note.[42]

The plural of ἕν: grammatical remarks

As to the formal correctness of the plural of 'one', we can start by considering the compound οὐδ(μηδ)-είς. In the *corpus* of authors writing in Attic, the plural forms οὐδ-ένες, μηδ-ένες (implying a plural ἕνες for εἷς: οὐδ', μηδ' ἕνες), οὐδένων, μηδένων (οὐδ', μηδ' ἑνῶν), οὐδέσι, μηδέσι (οὐδ', μηδ' ἑσί), οὐδένας, μηδένας (οὐδ', μηδ' ἕνας) were currently used.[43] To be sure, the parallel between εἷς and οὐδείς is only a partial one, as recognized by the late grammarians and lexicographers even if on the basis of a limited lexicographical record: they pretended that one of the main differences is that εἷς does not admit of plural forms.[44] We have seen that this is not the case;

[40] On the other hand, one finds only 9 occurrences of the adjective ἑναδικός, scattered among several authors.

[41] Forms of the plural genitive can be found at Proclus *In R.* I 258.16–18 Kroll (*bis*), *In Ti.* II 143.1 Diehl, and Damascius *Pr.* I 55.12, 71.11, II 44.1 Westerink, *In Prm.* IV 83.24, 90.22, 134.4 Westerink. The several occurrences in [Alexander] (*immo* Michael of Ephesus) *In Metaph.* 765.14–17 Hayduck, and Simplicius *In Ph.* 504.34–35, 505.15, are all included in comments on the first and third Aristotelian passages given at nn.17–18 above.

[42] Note also the 861 occurrences of ἑνότης, mainly in writings on theological subject. A corresponding adjective *ἑνοτικός is not attested.

[43] See Photius *Lex.* O 612 (III 119 Theodoridis) s.v. οὐδένες: τὸ πληθυντικὸν τοῦτο σύνηθες τοῖς παλαιοῖς· καὶ οὐδένων καὶ οὐδέσι καὶ οὐδένας. In the *corpus*, I did not find occurrences of plural neuter.

[44] See Choeroboscus *In Theod.* IV.1 205.13–25 Hilgard (differences between εἷς and the δείς of οὐ-δείς), in particular 205.20–23: τρίτον δὲ ὅτι τὸ μὲν εἷς οὐκ ἐπιδέχεται πληθυντικά· οὐδὲ γὰρ λέγομεν οἱ ἕνες (ἄτοπον γὰρ τὸν ἕνα πληθύνεσθαι)· τὸ δὲ δείς ἐπιδέχεται πληθυντικά· λέγομεν γὰρ οἱ οὐδένες τῶν οὐδένων τοῖς οὐδέσι. We find the same in *Etym.Magn.* 305.9–17 Kallierges, in particular 305.13–15: τὸ μὲν εἷς οὔτε δυϊκὰ ἔχει, οὔτε πληθυντικά· τὸ δὲ οὐδεὶς καὶ δυϊκὰ ἔχει καὶ πληθυντικά, οἱ οὐδένες, τῶν οὐδένων. The same opinion is given by Theon of Smyrna, *Exp.* 21.13–14 (cf. n.34 above): ἓν γάρ ἐστι καὶ οὐ πολλά, διὸ καὶ ἑνικῶς καλεῖται ἕν.

our information concerning the neuter ἕν can be summarized in the following terms:

1) Morphology: the plural of ἕν, ἑνός is ἕνα, ἑνῶν, as expressly said by [Alexander] when commenting on Aristotle *Metaph.* 1083a21–27 (n.18 above).[45] Also everyday Attic, for expressiveness' sake, employed the resource of the plural of 'one' in specific phrases.[46]

2) Accentuation: ἕν is monosyllabic, hence plural ἕνα, ἑνῶν (dative *ἑσί, unattested).[47]

[45] [Alexander] (Michael of Ephesus) *In Metaph.* 765.14–16, *ad* τῶν ἑνῶν of Arist. *Metaph.* 1083a25: τουτέστι τῶν μονάδων (τὸ γὰρ ‹ἑνῶν› ἐκ τοῦ ἕν, ἑνός, ἡ εὐθεῖα τῶν πληθυντικῶν τὰ ἕνα καὶ ἡ γενικὴ τῶν πληθυντικῶν τῶν ἑνῶν εἴληπται).

[46] *Suda* O 830 (III 581.11–12 Adler = Aelius Dionysius Ἀττικὰ ὀνόματα O 38 Erbse = Photius *Lex.* O 624 [III 121] s.v. οὐδ' ὑφ' ἕνων): Ἀττικοὶ ἀντὶ τοῦ ὑπ' οὐδένων λέγουσιν ὑπερβιβάζοντες. The hyperbaton refers to the deplacement of the preposition ὑπό. It is tempting to correct the accent of ἕνων to ἑνῶν, since in this way the *witz* is more effective (see also the following note): as it were, a hyperbaton from "by no-one" to "not by ones" and not to "not by one."

[47] But the manuscript tradition hands down ἕνων as a plural genitive at Proclus *In Ti.* II 147.1.6 and Damascius *Pr.* I 71.11 (the latter is corrected to ἑνῶν by the editor). J. Combès, in a note (at 152) *ad* Damascius *Pr.* I 55.12, somewhat rashly asserts: "Inutile de dire que le choix est arbitraire, puisque Damascius n'écrivait pas les accents, et que la forme n'existait pas dans la langue vivante." Of course, the choice is not arbitrary: there were accentuation rules in Greek antiquity—it suffices to think of Herodian's (late second century) Καθολικὴ προσῳδία—and the change from pitch accent to stress accent that was about to be accomplished in Diophantus' time made even more urgent the need to fix precise accentuation rules. Another matter is the evidence about *written* accents contained in late-antique papyri or in early (majuscule) Byzantine manuscripts—the earliest dated MSS. displaying a complete system of accentuation are *Vat.Barb.gr.* 336 and *Vat.gr.* 1666, both ca. 800. Now, given the highly technical character of our text, it is quite pointless to refer to the 'practice' (in fact, quite a variable one) of accentuation in literary papyri, a practice almost exclusively limited to poetic texts—see however C. M. Mazzucchi, "Sul sistema di accentazione dei testi greci in età romana e bizantina," *Aegyptus* 59 (1979) 145–167—what matters is what we find in technical texts. Important mathematical papyri such as the first-century *P.Vindob.gr.* inv. 19996 (PM^3 2322: metrological problems)

It seems thus clear that, from the point of view of everyday grammatical and linguistic competence, the neuter plurals ἕνα, ἑνῶν were considered unproblematic and in fact the most natural forms. Also for this reason, a form such as ἑνῆ can hardly be viewed as a plural of ἕν, as Tannery tentatively had it.

Yet, the form ἑνῆ in Diophantus is striking both in its morphology and in its accent. One might be tempted to correct the text, and there are at least two possibilities: correcting to ἕνα or postulating a different nominal form. As for the first possibility, in the majuscule grapheme ENH, -H can hardly be a misreading of -A (so ENH ← ENA, i.e. ἕνα): a technical text like the *Arithmetica* had every chance of being transcribed in an informal hand such as the one we see in the *fragmentum mathematicum bobiense*,[48] and there H is quite dissimilar from A. The grapheme -NHOIN- might instead be a palindromic quibble arising from an aural mistake of a late Greek speaker. The transliterator, who very likely knew the form ἕνα, could well have introduced it, thus correcting to ENA (i.e. ἕνα) the presumed mistake. He could have done this but he did not. There must have been some hard thought on his part about this stretch of text: the

and the early second-century *P.Mich*. III 144 (arithmetical problems very much in the style of Diophantus) are not accented, nor are the astronomical papyri *P.Oxy*. 4133–4300a edited in A. Jones, *Astronomical Papyri from Oxyrhynchus* (Philadelphia 1999). The 5th–6th century *fragmentum mathematicum bobiense* (see n.48) is replete of abbreviations but bears no accents; even in the early ninth-century majuscule *Par.gr*. 2389 (Ptolemy *Almagestum*), where no abbreviations are used, the accents were (selectively) added by a later hand. My choice in the next paragraph of assigning the accentuation of Diophantus' *Arithmetica* to the medieval transliterator from a majuscule to a minuscule script is arbitrary but is likely to strike near the truth: the accents, in Proclus' and Damascius' passages mentioned at the beginning of this note as well as in the case of Diophantus' ἑνῆ, are unlikely to have ancient authority.

[48] This is the *scriptio inferior*, dated to the 5th–6th century, of the palimpsest *Ambros*. L 99 sup. (Isidore of Seville *Etymologiae*), whose pages 113–114 are reproduced in C. Belger, "Ein neues Fragmentum mathematicum Bobiense," *Hermes* 16 (1881) 261–284.

subsequent ἑνός and the immediately preceding δύο (note the [deliberate?] oxymoron δύο [*immo* β΄] ἑνῆ) suggested to him to isolate the grapheme ENH so as to make it a word on its own and to provide it with a rough breathing, but only his grammatical skills made him put a circumflex above the H instead of resorting to the *lectio facilior* ἕνα.

As a second possibility we might postulate, as an analogical formation of sorts based on its opposite πλῆθος, a singular like *ἕνος, ἕνους, whose plural should be *ἕνη, ἑνῶν, ἕνεσι, and correct the text accordingly by simply changing the accent. Of course, this amounts to replacing one unattested form with another unattested form.

We have also seen that the reading ἑνῆ has a sound manuscript tradition: it would therefore be unmethodical to correct the text, in view also of the fact that the tradition is a highly technical one and that some questioning should have arisen about a form that does not coincide with those attested in the philosophical tradition (the plural ἕνα of ἕν and the plural forms of ἑνάς).

The most economical solution, in fact, is to assign ἑνῆ to an adjective ἑνής ἑνές, plural ἑνεῖς ἑνῆ, having the genitive plural ἑνῶν as an entry in common with the attested plural of ἕν and with the plausible plural of an unattested *ἕνος. The redactor of our text preferred this form to ἑνικός, of which he could have seen a model in ἑνάς > ἑναδικός, but that, in its turn, was charged with grammatical overtones. An adjective such as ἑνής ἑνές, instead, widens the semantic range of the numeral ἕν in the direction of an attributive, while keeping, in the very clause in which it is introduced, an exclusive link with the originating stem (ἑνῆ … ἐκ μὲν τοῦ ἑνὸς … ἐκ δὲ τοῦ ἑνὸς …): what is at issue are not simply "two units of wine," but "two singularities/peculiarities of wine," that is, two qualities of wine ("qualitates" in the pen of Tannery).

This discussion should not make us forget that a dual of 'one' is required in our passage, not a plural. The dual of ἑνής ἑνές

should be ἐνεῖ.[49] But then, we might well be entitled to correct the text, postulating a mistake of quality (most likely an itacism) on the part of some copyist. Against this hypothesis stands the simple fact that forms of dual are banned from the lexicon of Greek mathematics; in particular, no dual is attested in the *Arithmetica*, where the substantives or adjectives denoting or qualifying pairs of numbers are always in the plural.[50]

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[49] On the impossibility of forming such a dual (see n.44 above on the analogous impossibility, alleged by Choeroboscus, of forming the plural), see Choeroboscus *In Theod.* IV.1 132.34–38: πῶς γὰρ τὸ εἷς καὶ τὸ ἕν εἰς δυϊκὰ κλίνεσθαι δύνανται δυϊκῷ ἀριθμῷ θεωρηθῆναι μὴ δυνάμενα; ἐὰν γὰρ παραληφθῶσιν ἐν δυϊκῷ ἀριθμῷ, καὶ τοῦ σημαινομένου ἐκπίπτουσι καὶ οὐκ ἔτι σημαίνουσι τὸν ἕνα ἀριθμόν, ὥσπερ καὶ ἐκ τοῦ ἐναντίου ὁ δύο ἀριθμὸς οὐ δύναται ἑνικῷ ἀριθμῷ θεωρηθῆναι. On the "nominative singular" of δύο and τρία see n.17 above.

[50] This note owes very much to discussions with Alessandro Lami.