Petronian Numbers

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PLUTARCH remarks that a certain Petron of Himera declared the universe to consist of 183 *kosmoi* arranged so as to form a triangle with sixty worlds along each side and the remaining three at the corners (*De defectu oraculorum* 22.422B). This remarkable theory was known to Plutarch indirectly from the writings of Phanias, the Aristotelian scholar of Eresos in Lesbos, who quoted Hippys of Rhegion on Petron's cosmology.[1] The date of Hippys is not known—there is no special reason to date him as early as the first half of the fifth century B.C.[2]—but we need not doubt that Phanias reported a genuine piece of western Greek (or possibly Sikel)[3] philosophical doctrine. Petron's theory deserves closer examination.

Plutarch was puzzled by the statement that the *kosmoi* were *ἁπτομένους ἀλλήλων κατὰ στοιχεῖον* (422D), but he earlier states (422B) that they were in contact with each other and rotated "as in a dance," *ἅπτεσθαι δὲ τοὺς ἐφεξῆς ἀλλήλων ἀτρέμα περιιόντας ὥσπερ ἐν χορείᾳ.* The meaning therefore is that each world rotated on a fixed axis but touched its neighbour, which therefore rotated at the same speed but in the opposite sense, if all the *kosmoi* were of the same size. Plutarch's words suggest that the three *kosmoi*, one at each of the corners, also rotated, but here a problem would have demanded Petron's attention if he thought his system out thoroughly. Three spheres in contact, having parallel axes and equal diameters, cannot drive one another. The solution is to place spheres at the corners, each having a greater diameter than the two other spheres in contact with it. At each corner there would thus be two, not three, points of contact.

Petron's difficulties would not have ended there, however, for a continuous series of mutually driving, contra-rotating spheres must

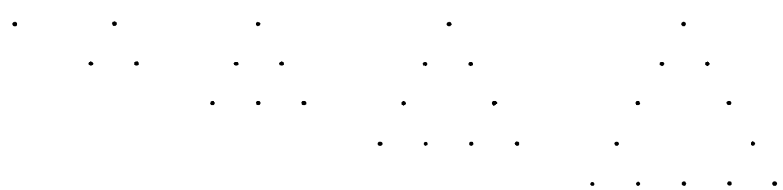
[1] *De def.or.* 23.422D.

[2] *Cf.* Jacoby on *FGrHist* 554 T 1 and F 5. The Suda *s.v.* *Ἵπ⟨π⟩υς Ῥηγῖνος* (Hippys T 1) state: *ἱστορικός, γεγονὼς ἐπὶ τῶν Περσικῶν. καὶ πρῶτος ἔγραψε τὰς Σικελικὰς πράξεις, ἃς ὕστερον Μύης ἐπετέμετο.*

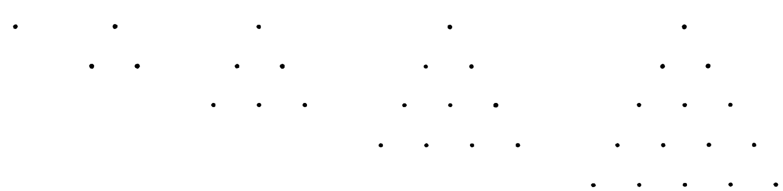
[3] E. A. Freeman, *The History of Sicily from the Earliest Times* II (Oxford 1891) 159–160.

be of an even number if one adjacent pair is not to rotate in the same sense and so create friction at the point of contact. Petron's system had 183 spheres if Phanias reported his theory correctly, and so if all the spheres were in contact, each with its neighbour, there must have been friction at one point. It is possible that Phanias or Hippys misunderstood Petron, who perhaps gave 61, not 62, spheres to each side of his triangular universe. The sum of spheres in it would then have been $(59\times 3)+3=180$, an even number, permitting a free running system.

Petron's system is of little astronomical interest, although it is remarkable that Leukippos seems also to have contemplated a triangular universe.[4] The Petronian triangle does, however, have a place in early Greek number theory, because it is different from normal Pythagorean triangular numbers. The typical Pythagorean number is a sum of successive terms in the series of natural numbers. Thus $1+2+3+\ldots+n=\frac{1}{2}n(n+1)$ is a Pythagorean triangular number of side n. The Pythagorean series begins 1, 3, 6, 10, 15 ... Petronian triangular numbers are of the form 1, 3, 6, 9, 12, 15 ... Thus



are Petronian triangles, but

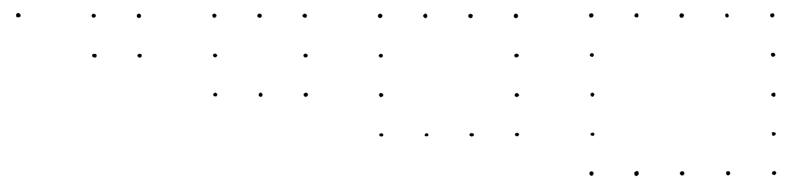


are Pythagorean. Erwin Schroedinger failed to understand this distinction when he wrote: "One of the early Pythagoreans, Petron, contended that there were altogether 183 worlds, arranged in a triangle, though, by the way, this is not a triangular number."[5] The

[4] Diels, *Vorsokr.*^{11} 67 A 24 line 33.

[5] *Nature and the Greeks* (Cambridge 1954) 36. W. Nestle, *RE* 37 (1937) 1191 *s.v.* PETRON, states that there were 163 Petronian *κόσμοι*, but that seems to be a mistake.

corresponding Petronian square numbers would be of the form:



So this series would begin 1, 4, 8, 12, 16 . . .

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