What to Say to a Geometer

Michael J. White

When Epicurus (D.L. 10.6) advised Pythocles to "flee all *paideia*," it seems extremely likely that education in the established geometrical science was part of what he had in mind. Indeed, Epicurean aversion to the geometers' *pulvis eruditus* eventually became something of a rhetorical commonplace. Cicero (*Nat.D.* 2.47f) comments on this aversion and the consequent mathematical ignorance of Epicureans, attributing to it, among other deficiencies, the aesthetic blindness that renders them incapable of recognizing that a sphere is more beautiful than a cone, cylinder, or pyramid. While there were some Epicureans who were—or previously had been—mathematicians,[1] it seems that acceptance of the Epicurean world-view generally involved, for these individuals, a conversion *from* the practice of geometry. Polyaenus, an eminent 'first-generation' Epicurean, who was perhaps the most distinguished of the converted mathematicians, is described by Cicero (*Acad.* 2.106) as having come to believe that "all geometry is false" after he had accepted the views of Epicurus. Zeno of Sidon apparently was an exceptional figure who, according to the account of Proclus, seems to have continued his mathematical work as an Epicurean, arguing that (all? many? some?) theorems of geometry do not follow without some additions to the (normally accepted?) set of postulates.[2]

Despite a few problematic cases such as that of Zeno, it seems there is good reason to agree with David Sedley that "the wholesale rejection of geometry was still orthodox Epicureanism" (24). Even 'wholesale rejection' can come in several varieties, however. Wholesale rejection can be grounded in ignorance, prejudice, or prepossession. And of course (witness Cicero) the Epicurean rejection of

[1] See the summary account in D. Sedley, "Epicurus and the Mathematicians of Cyzicus," *ChronErcol* 6 (1976 [hereafter 'Sedley']) 2–26.

[2] Proclus, *In primum Euclidis elementorum commentarium*, ed. G. Friedlein (Leipzig 1873) 199.

geometry was often so interpreted. But wholesale rejection can also be based on undestanding and prudence. A rejection of geometry —as geometry had developed in antiquity—is an intelligent and prudent response (perhaps the *only* viable response) for a philosopher seriously committed to the doctrine of indivisible quanta of magnitude. For such a philosopher, the best stratagem available in antiquity probably would have been an entirely defensive stonewall: to claim that geometrical concepts simply are not applicable to such quanta, and consequently to refuse to pursue geometrical arguments pertaining to them.

When one introduces indivisible quanta of magnitude, in the true sense of 'quanta' (*i.e.*, the sense connoting 'positive measure' or bulk), protestations in the form of pointed geometrical questions are almost certain to be voiced. What is the shape of such a quantum? (Assumption: any shape implies a geometrical organization of spatial proper parts, which contradicts the notion of a conceptually or theoretically indivisible spatial magnitude.) What are the dimensions of such a quantum? That is, how far is it from one side of it to another? (Assumption: any positive distance, specified in terms of some real-valued measure, means that we should, *theoretically* or *conceptually*, be able to talk about half that distance, a quarter of that distance, *etc.*—even if, *as a matter of fact*, there are no separable bodies or particles that small.) Probably the most effective way of dealing with such questions is to deny their applicability to indivisible quanta. The questions only arise, it might plausibly be maintained, because the questioner mistakenly persists in (tacitly or explicitly) conceiving the quanta as embedded in a matrix or medium that is infinitely divisible and continuous. To what extent did ancient proponents of the quantum model of magnitude make use of this strategy of rebuttal to geometrical criticism of their doctrine? The evidence is scanty. One apparently early and apparently non-Epicurean example comes from the pseudo-Aristotelian treatise *On Individual Lines* (968b15–17), where the classification 'rational/irrational' is withheld from lines constructed from a supposedly minimal two-dimensional figure.

An Epicurean example that has been much discussed comes from the *Letter to Herodotus* 58f. In this passage, Epicurus appears to conclude analogically (from the case of perceptible minima) that *minimae partes* within an atom are arranged successively or discretely but are not contiguous and that, by their number, they determine the bulk or size of the whole atom. Epicurus is not obliged to answer—and he wisely resists answering—questions that arise because of the tacit assumption of an infinitely divisible and continuous spatial matrix in which the quanta are embedded: what are the shapes of the minima? how, precisely, can they be successively ordered without touching? are there interstices between them? *etc.*

It seems clear that wholesale rejection of geometry can come in less- and more-sophisticated varieties. But there are other possibilities with respect to the relation between Epicurean physics and geometry. Vlastos has, I believe, adopted a much too restrictive assumption with respect to this matter. He maintains, in effect, that mathematically interested and educated Epicureans had only two options. The first is the development of a finitistic geometry in the rather narrow sense of a postulate set, analogous to that of Euclid but with an intended model of discrete elements. The second option, which arises in the absence of such an axiomatic development of finitistic geometry, is the "consign[ment of] the whole of geometry to the devil"[3] or the acceptance of the developing 'Euclidean' geometry with its *mathematical* assumptions of the continuity and infinite divisibility of magnitude.

Vlastos is surely correct in claiming that there is no evidence for the existence of any ancient postulate set the intended model of which contained discretely ordered (and, at least for bounded constructions, a finite number of) *Urelemente.* It is not surprising that there should be no evidence of ancient axiomatic development of such a finitistic geometry. Vlastos comments, quite reasonably, "I find it very hard to see what the finitist geometry ... would be like" (127). In contemporary mathematics, such finite models typically arise in the algebraic study of 'absolute' geometry, in which fundamental concepts that are very abstract from the physical point of view replace the more intuitively 'comfortable' concepts of traditional elementary geometry.[4] I believe that Vlastos is on firm ground in pointing to the *a priori* implausibility of the ancient axiomatic development of finitistic geometry: and I agree with Sed-

[3] G. VLASTOS, "Minimal Parts in Epicurean Atomism," *Isis* 56 (1965 [hereafter 'Vlastos']) 127.

[4] Absolute geometry pertains to geometry without anything corresponding to the Euclidean parallel postulate. See *e.g.* H. Wolff and A. Bauer, "Absolute Geometry," in H. Behnke *et al.*, edd., *Fundamentals of Mathematics*, tr. S. H. Gould (Cambridge [Mass.]/London 1974) II 129–73.

ley (26 n.24) that Vlastos "spells out a strong *argumentum e silentio* against the existence of a special Epicurean geometry."

In fact, there seems to be no serious opposition to Vlastos on this point. Although Jürgen Mau has sometimes been cast in the rôle of a defender of the idea of a 'special Epicurean mathematics', the claims that he advances in his paper on that topic are actually quite modest: (a) the Epicurean rejection of εἰς ἄπειρον τομή is the rejection of "the mathematical axiom that any size can be bisected again and again *ad infinitum*," and, consequently, the Epicurean ἐλάχιστον or minimal quantum is, in some sense, *geometrically* minimal and indivisible; (b) from the Epicurean standpoint, "reasoning without the axiom of division *ad infinitum* is legitimate because from the reliable testimony of Archimedes [Mau has in mind the Μέθοδος] we know that it is possible to find new and true theorems without that axiom."[5]

With respect to the second option, perhaps some mathematically sophisticated converts to Epicureanism did choose to consign geometry to the devil. Cicero's references to Polyaenus may plausibly be interpreted as implying something of the aversion, on Polyaenus' part, of the reformed smoker to cigarettes. But there were doubtless other reactions on the part of Epicurean mathematical *cognoscenti*. As Vlastos notes, Demetrius of Laconia wrote a treatise on geometry; and Zeno, mentioned above, seems to have pursued mathematical matters (as an Epicurean) with a diligence sufficient to elicit a book-length response from Posidonius.[6]

Did any of these mathematical Epicureans work within the tradition of Euclidean geometry, perhaps contributing to its development? Vlastos offers as an instance an Epicurean Zeno of Sidon.[7] I believe, however, that the plausibility of Vlastos' view of Zeno's mathematical endeavors largely rests upon Vlastos' claim (135–47) that Epicurean *minimae partes* are not geometrically or mathematically indivisible quanta.

Vlastos' interpretation of *minima* is summarized by a proposition,

[5] J. Mau, "Was There a Special Epicurean Mathematics?" in E. N. Lee *et al.*, edd., *Exegesis and Argument: Studies in Greek Philosophy Presented to Gregory Vlastos* (Assen 1973) 422, 428, 429.

[6] Vlastos 127 n.35; Procl. *In Eucl.* 199f.

[7] G. Vlastos, "Zeno of Sidon as a Critic of Euclid," in L. Wallach, ed., *The Classical Tradition: Literary and Historical Studies in Honor of Harry Caplan* (Ithaca 1966) 148f.

characterized by him as a "law of nature" and a "physical statement about the atoms" (designated L[I]): atoms are so constituted that variations in atomic lengths occur only in integral multiples of the smallest atomic length (138). With respect to L(I), Vlastos comments (147) that "if this were the law of nature it is meant to be, it should be compatible with any (self-consistent) mathematical system." But would it be? In order to avoid inconsistency, the notion of variations in atomic lengths must be interpreted in such a restricted, artificial sense that L(I) becomes virtually vacuous. First, it is important to note that the smallest atomic length, designated q by Vlastos, does not represent any dimension of the parts into which atoms may be physically divided because all atoms are *qua* atoms physically indivisible. Whether the dimensions or atomic lengths of a particular atom are all multiples of q depends on what we count as 'dimensions' or 'atomic lengths'. For example, if the edges of a cubic atom count as dimensions and are each of atomic length $2q$, then the diagonal of the cube cannot count as a dimension/atomic length, for it will not be commensurable with the lengths of the edges of the cube. But we could just as well count the diagonal as a dimension/atomic length—by an appropriate *geometrical* construction of the cube—in which case its edges cannot be atomic lengths. Could we rather arbitrarily limit the concept of atomic lengths to the edges of solids? Not unless we want to rule out a variety of regular solids as possible atomic shapes. For a regular pyramid with square base of side $2q$ and height $2q$ will have edges with lengths incommensurable with q. Vlastos suggests that we might try to preserve q as minimal atomic length by supposing that "all atoms, no matter how irregular might be their contours, could be (theoretically) broken down in the last analysis into parts of parallelepipedal shape, and the atomic lengths of the whole atom would be the set composed of all the atomic lengths of these component parts" (138 n.86). Presumably, the "atomic lengths of these component parts" refers to the lengths of the *sides* of the parallelepipeds, which will be mutltiples of q. But the problem is that, since we are referring to a *theoretical* 'breaking down' rather than a physical one and since the theory in question is, according to Vlastos' assumption, Euclidean geometry, there are any number of ways of geometrically breaking down or carving up such atoms. Why, for example, think of a cube as geometrically constituted of smaller cubes with edges commensurable in length to the edges of the larger cube rather than as geometrically constituted of pyramids, which have edges not commensurable in length with the edges of the larger cube? And why think of atomic lengths as *sides* of parallelepipeds rather than, say, their *heights* (which may not be commensurable with their sides)? There do not seem to be any reasons, other than arbitrary geometrical stipulation, for ruling out geometrical 'decompositions' of atoms that do not yield 'parts' having edges the lengths of which are all multiples of some q. But when we invoke such geometrical stipulations, we certainly appear to be placing *some* sort of limitation on Euclidean geometry.

It might be granted to Vlastos that the Epicurean doctrine that atoms are found only in a limited variety of sizes and shapes is a 'law of nature' or 'physical statement' without mathematical import. But I cannot see how restrictions on how we are to conceptualize, geometrically, the structure of atoms—which are *ex hypothesi* physically indivisible—can be anything but geometrical restrictions; and these restrictions must appear entirely arbitrary if we are assuming that Euclidean geometry characterizes the spatial matrix of atomic solids.

Other criticisms of Vlastos' interpretation of Epicurean minimal quanta have been given. Furley points out, for example, that since Vlastos' interpretation does not attribute to the Epicureans the doctrine of minimal, discrete *spatial* distances, an infinite number of atomic shapes could easily be produced by slight spatial reorientation of the finite number of parallelepipedal solids into which each atom is properly (geometrically) analyzed, according to Vlastos. In order to preclude an infinite variety of atomic shapes, then, Vlastos needs another principle (geometrical or physical?) restricting possible 'geometrical conjunctions' of his canonical parallelepidedal solid atomic parts.[8] Furthermore, Sedley notes (26 n.26) that the "mathematical fragments of Demetrius of Laconia make it appear that the ἐλάχιστον posed a threat to geometry; which on Vlastos' interpretation it would not do." In sum, then, I think that there is reason to reject Vlastos' contentions that L(I), *qua* 'physical statement', captures all that the Epicureans intended by talk of 'least' and 'partless' magnitudes and that mathematically sophisticated but 'physically orthodox' Epicureans need not have had any *fundamental* objections to Euclidean geometry. At the very least, this

[8] D. J. Furley, *Two Studies in the Greek Atomists* (Princeton 1967) 42f.

contention would seem to need to be supported by a variety of stipulations and restrictions, which appear *ad hoc* and which are not obviously physical, rather than mathematical, in character.

It is easy to see, however, that so long as Vlastos' interpretation of the minimum stands, there is some considerable reason to maintain that Zeno's criticisms of Euclidean geometry must have been constructive. For example, according to Vlastos' interpretation of the doctrine of the minimum, Zeno would have no obvious *mathematical* objection to the additional assumption that (according to Procl. *In Eucl.* 214.21f) he considers necessary in order to construct an equilateral triangle (the first proposition of the first book of Euclid). Zeno argues that in order for the construction to be legitimate, it must be assumed that two lines do not share a common segment. According to Vlastos' interpretation, the Epicurean doctrine of the minimum could not constitute the basis of a denial of the existence of lines, points, surfaces, in the 'true limit' sense, nor the denial of a *mathematical* conception of divisibility *ad infinitum*. If, however, Vlastos' interpretation is rejected—if, in other words, the doctrine of spatial minima was understood as having geometrical significance—it becomes very difficult to see Zeno as a basically constructive (although perhaps niggling and not very penetrating) critic of the Euclidean axiomatization of geometry. If Zeno subscribed to a doctrine of indivisible quanta of magnitude, it seems likely that, as Sedley says, "Zeno regarded the additional premise as false," indeed, *mathematically* false (25). Since a line cannot be unextended in one dimension (because there are no such limit entities according to a *geometrical* doctrine of indivisible quanta), it is entirely possible—indeed necessary—for two nonparallel straight lines to have a common segment. To show that a given theory requires, in order to derive its theorems, a postulate that one takes to be false—or that the theory entails a false proposition—can easily be presented as a quite destructive criticism of the theory: a *reductio* of the theory, in fact.

I very much suspect that such was the nature of Zeno's criticism of Euclidean geometry. But where does this leave Zeno and other Epicurean mathematicians? If we accept Vlastos' exhaustive dichotomy of adherence to Euclid or the consignment of geometry *in toto* to the devil, the answer is obvious. But there are other options. Sedley mentions a sort of piecemeal, non-axiomatic "applied geometry," the exchange of a mathematical science for "an inexact but serviceable discipline" (26). And I believe that Mau perhaps entertains the same hypothesis. Another possibility, not inconsistent with this hypothesis, is the dialectical use of geometry. By 'dialectical use' I mean the criticism of Euclidean geometry not necessarily as an end in itself but as a tool in the development of atomistic physical doctrines. Specific evidence for such an undertaking is, alas, scanty to say the least. But in what follows I develop a speculative 'plausible story' of Epicurean criticism of Euclid's parallel postulate as a means toward clarifying the doctrine of the παρέγκλισις. I quite realize that I am skating on very thin ice indeed, in terms of actual history of Epicurean thought. But I hope that my story, even if suspect as an historical hypothesis, has some *conceptual* interest as an example of a way in which the mathematically sophisticated Epicurean *could* have used geometry to his own purposes.

In his discussion of Euclid's parallel postulate (1.5), Proclus (*In Eucl.* 368.27–369.1) reports an argument that purports to establish the *contrary* of the parallel postulate, *i.e.*, that "it is impossible that lines produced at angles less than two right angles should meet." The argument is the following (see *fig.* 1). Take two straight lines AB

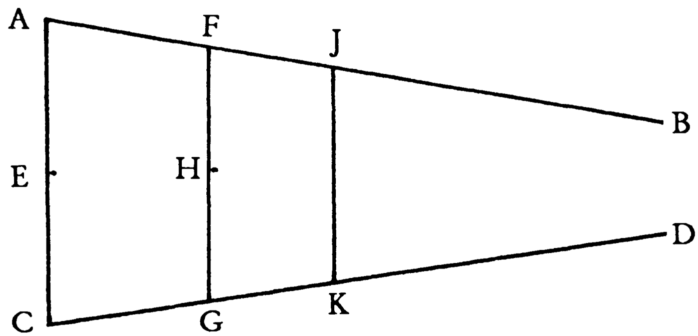


Figure 1

and CD and a straight line segment AC connecting them in such a way that the sum of the interior angles ("on the right") is less than two right angles. Bisect AC at E and measure off a length AF equal to AE on AB and a length CG equal to CE on CD. It is clear that AB and CD do not meet at F and G (*i.e.*, it is clear that F and G are not, in fact, the same point). For if AB and CD did so meet, the sum of two sides of a triangle (*i.e.*, of AF and CG) would be equal to the third side AC, "which is impossible." So FG must be a line segment having some length. Bisect it at H; and again measure out line segment FJ on AB equal to FH and line segment GK on CD equal to GH. The same kind of argument can be used to show that lines AB and CD do not meet at points J and K. Since this process can be continued infinitely, which results in taking further and further segments on AB and CD, the argument's proponents conclude (according to Proclus 369.1–20) that the straight lines do not meet anywhere.

In his discussion of this argument, Proclus makes a promising beginning (369.21–370.2):

> Although [the proponents of the argument] speak the truth, they do not say as much as they believe. That it is not possible to define the point of intersection in this straightforward way is true. However, it is not true that the lines do not meet at all.

The argument provides a method of generating a sequence of pairs of points on AB and CD, respectively, and shows that, for each of these pairs, the members of the pair cannot coincide. However, as Proclus claims, it does not follow from this construction that the lines do not intersect. There is the case where the two angles (CAB and ACD) are not equal and the point of intersection falls *between* successive points on one of the lines (see *fig.* 2). Proclus *may* have

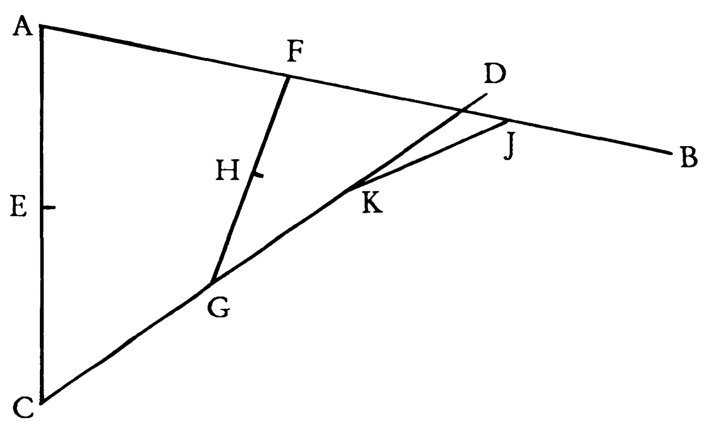


Figure 2

had this case in mind in the first part of his more detailed response to the argument (370.2–10). But this part of his response seems quite confused, and I shall not go into details. In general, after his promising initial comment quoted above, Proclus' analysis of the argument is disappointing.[9]

Another case that I wish to consider more closely is that in which angles CAB and ACD are equal. Here, according to the parallel postulate, line segment AC is the base of an isosceles or equilateral triangle having as (equal-length) sides segments of lines AB and CD. In this case the sequence of points A, F, J ... (on line AB) and the sequence C, G, K ... (on line CD) converge to the intersection of the lines (*i.e.*, the vertex of the triangle) *as a limit of both sequences.* For each n, the pair of points which are the n–th members of each sequence are some finite distance ε from the point of intersection; but for any distance ε, there is a natural number N such that for all $n > N$, the distance between each of the n–th members and the point of intersection is less than ε. When Heath notes a certain similarity between this criticism of the parallel postulate and Zeno's paradox of Achilles and the tortoise, he is evidently thinking of this particular case.[10] Although the construction on lines AB and CD can be continued *ad infinitum*, the sequences of points resulting from the continued construction converge to a point finitely distant from A and from C.

Proclus does not indicate the provenance of this argument, and, so far as I know, there is no evidence that would come close to deciding the issue. However, an interesting hypothesis, which I shall entertain, is that the argument is an Epicurean one. The one explicit principle employed in the argument that is mentioned by Proclus is Euclid 1.20 to the effect that the sum of any two sides of a triangle is greater than the remaining side. Proclus reports that

[9] After the opaque argument at 370.2–10, Proclus states the obvious, that the argument proves too much, noting that it will be refuted if a line can be drawn from A to G. But if it cannot, the first as well as the fifth postulate of the first book of Euclid will be contradicted. Finally, Proclus comments that "someone could say" (evidently, someone not assuming the parallel postulate) that straight lines whose interior angles total less than two right angles but are greater than or equal to some angle a "remain nonsecant"; but if the total is less than a, they intersect (370.4–10). If one lets one of the interior angles remain a right angle and thinks of a as the angle of rotation of the other line, less than a right angle, at which the lines become nonsecant, a is the 'angle of parallelism' in Lobachevskian geometry, as noted by T. L. Heath, *The Thirteen Books of Euclid's Elements*² I (New York 1956) 207.

[10] Heath (*supra* n.9) 206.

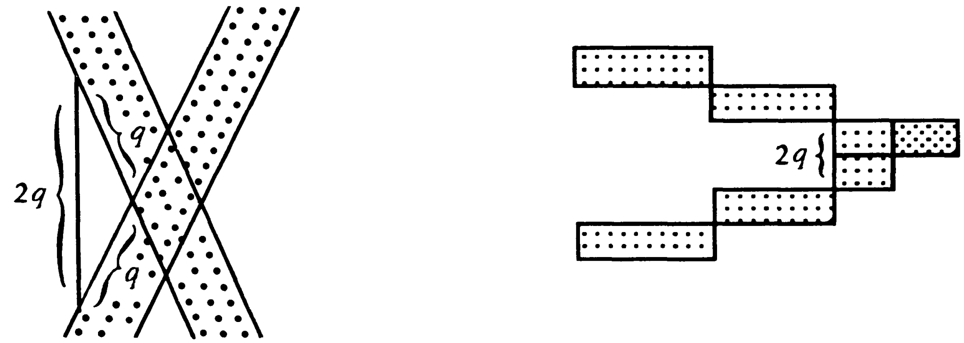
Epicureans used this theorem as an example of the inutility of geometry because the proposition is "clear even to an ass and requires no demonstration"—*i.e.*, the proposition belongs among τὰ ἐμφανῆ, 'things that are manifest'.[11]

It will be noted that the rebuttal of the argument for the case we are considering, in which the two angles are *equal*, depends upon the infinite divisibility of magnitude. In effect, the construction of the argument yields a sequence of ever smaller similar isosceles or equilateral triangles, the limit of whose areas is zero. It is perhaps worthwhile to contemplate the argument within the context of the assumption of indivisible quanta of magnitude. Let us suppose, with Euclid, that two lines (whose interior angles in relation to an intersecting line are equal and less than 180 degrees) do intersect on the same side, forming an acute angle. Now, consider a position on each line when the two lines are two minimal units ($2q$) apart. Bisecting this interval, we measure off a further minimal distance q on each line and ask if the lines intersect at the distance of q 'to the right' of the positions we are considering. The answer will have to be 'no' because, were it 'yes', we would apparently have a triangle with base ($2q$) equal to the sum of its sides (each of minimal length q). Since the lines cannot intersect in anything less than a minimal distance q, two possibilities seem to remain: (a) the lines do not intersect at all; (b) each line shifts or 'jumps' (discontinuously) one minimal unit toward the other. So, on the assumption that lines have a breadth of minimal magnitude q, they will then be contiguous for a distance of q or some multiple of q, and then they will coincide for a distance of q or some multiple of q.[12] With respect to

[11] Procl. *In Eucl.* 322.5–8; *cf.* 322.10–14 for the Epicurean claim that a hungry ass will take a straight line to fodder rather than, so to speak, 'triangulating' to it. This is an ancient variety of argument for a self-evident truth that I like to call 'birds b'lieve it; bees b'lieve it; even educated fleas b'lieve it'. There is another noteworthy example at Sext. Emp. *Pyr.* 1.69, where the Stoic Chrysippus is presented as arguing that even a dog accepts a generalizaton of *modus tollendo ponens*: either A or B or C; but not A and not B; therefore, C. For, according to Chrysippus, a dog coming to a triple fork in the road and not detecting the scent of its quarry in two of the three paths will set off straightway down the one remaining path without bothering to check for the scent.

[12] Sedley suggests to me a third possibility: the 'straight' lines are conceived as contiguous for (some multiple of) q and, then, as 'changing positions' each with the other and remaining contiguous with each other for (some multiple of) q before diverging. Would this conception of each line 'quantum shifting *over* or *past*

alternative (b), one 'straight line' will evidently have to shift to the position of the other: they cannot meet 'halfway', as it were, because that would involve a shift of $1/2q$ of each towards the other. Which line is conceived as shifting and which is conceived as 'staying in position' seems an entirely arbitrary matter, from the geometrical perspective (see *fig.* 3).



The sort of intersection of lines ruled out by Euclid 1.20

'Quantum shift' of straight lines

Figure 3

What might one conclude from the 'Epicurean' argument that I have been imagining? First, it provides an argument that *if* some version of the parallel postulate is to be retained within the context of mathematical atomism, straight lines that are not parallel must have a common segment. The import of this claim is more than the fairly obvious point that, if lines are conceived as having some (minimal) magnitude, their intersection or 'crossing' must have some magnitude. The argument suggests that the common segment shared by intersecting straight lines is achieved by a 'bending' of one or other of the lines. But it is not a bending in the sense of con-

---

the other', so to speak, constitute a counterexample to the conclusion that straight lines must either (a) share a common segment or (b) not intersect? The answer depends, I suppose, on whether one takes the 'quantum shift over/past each other' to be an instance of 'intersecting' or 'meeting one another' (συμπιπτοῦσιν ἀλλήλαις). If this identification is made, then this situation obviously does constitute a counterexample. But the identification need not, I think, necessarily be made, for in this situation there is no *part*, whether segment or 'limit' (*e.g.* point), at which the two lines intersect.

tinuous curving of the line; rather it is a discontinuous quantum shift or 'jump' of the line. Such a proof would certainly give added force to the complaint of Zeno of Sidon that Euclid's construction of an equilateral triangle (1.1) assumes that straight lines do not have any common τμήματα (segments). If the parallel postulate *requires*, according to atomist lights, common segments of nonparallel straight lines while Euclid's proposition 1.1 requires the contrary of the assumption, it is perhaps easier to see why Proclus (*In Eucl.* 214.15–17) should categorize Zeno among those who, in raising objections to this proposition, "believed that they were refuting the whole of geometry."

Such an argument concerning the parallel postulate could also have been useful in developing the Epicurean doctrine of the atomic swerve. Lucretius (2.244f) notes that the atomic παρέγκλισις must not be "more than a minimum lest we seem to suppose oblique motions, and that the truth of the matter refutes." Concerning the apparent distinction drawn in this passage between the swerve and oblique motion, Sedley suggests that an atom moving in a straight line "can shift sideways by one ἐλάχιστον without leaving that line" (25). My suggestion is that Epicureans could have derived support for this most un-Euclidean doctrine from an atomistic criticism of the parallel postulate derived from the argument reported by Proclus. The idea is that the atomistic criticism of the parallel postulate shows that—at least in the case of nonparallel straight lines—a line can make such a 'quantum shift' *itself* and remain the same straight line.[13] Of course, that the argument's source was Epicurean and that they made this sort of dialectical use of it is but a conjecture, supported by very little circumstantial evidence.[14]

[13] Sedley points out that in the case of 'natural' atomic motion and the παρέγκλισις, we are evidently dealing with parallel straight lines with a privileged orientation (*i.e.*, down/up). And it is not clear that we would have reason to think of *these* lines as undergoing quantum shifts. Particularly if we do, it seems clear that the notion of parallelism will need to be reconsidered. As Michael Ferejohn expressed it to me, parallel lines will have to be far enough apart to prevent them from coinciding due to quantum shifts. Of course, it is possible simply to think of parallel lines in the privileged orientation as being *defined* by the motion of distinct atoms 'in free fall'—with or without a minimal swerve.

[14] I am indebted to Paul Vander Waerdt for bringing to my attention one additional piece of circumstantial evidence: *P.Herc.* 1005.7 attributes to Zeno of Sidon a Περὶ παρεγκλίσεως καὶ τῆς τοῦ ἀθρόου προκαταρχῆς (*On the Swerve and the*

It does, however, seem plausible that Lucretius' deprecation of oblique atomic motion is indicative of a notable insight on the part of mathematically sophisticated Epicureans: a doctrine of geometrically minimal *linear* magnitude will not leave unscathed the Euclidean doctrine of continuous and infinitely divisible *angular* magnitude. Consequently, the doctrine of minima will affect the angular as well as the linear components of atomic motion. A criticism of Euclidean geometry, such as the one we have been discussing, could from the perspective of a conception of indivisible quanta of magnitude certainly help to sharpen such an insight.

In conclusion, I return to the title of this essay. What could the ancient proponent of a theory of indivisible quanta of magnitude say to a geometer? There are, I think, three possibilities. (a) He could devise a competing, axiomatic geometry consistent with his own theory, a finitistic geometry the intended model of which contains discrete *Urelemente*. As Vlastos has noted, there are absolutely no signs of such an ancient endeavor. And the *a priori* plausibility that there was such an undertaking is slight. (b) The proponent of indivisible quanta could assume an entirely defensive stance, refusing to consider the application of geometrical arguments to this physical theory—either (i) because of ignorance of or aversion to geometry or (ii) because of the conviction that any such argument will ultimately beg the question by tacitly assuming that the quanta are embedded in an infinitely divisible and continuous spatial 'matrix'.

Finally, (c) our proponent of indivisible minima can attempt to engage the geometers dialectically. Here, too, distinctions need to be drawn. The strongest form of dialectical argument is the *reductio ad absurdum*: in this case, it would be an attempt to show that the developing standard geometry, which assumes the infinite divisibility and continuity of spatial magnitude, is inconsistent or (more specifically) actually entails the existence of indivisible quanta of magnitude. The one clear historical example of this tack, which I have not discussed, is an argument reported in the pseudo-Aristotelian *On Indivisible Lines*, which purports to derive the existence of such minima from the geometrical concept of com-

---

Origin of the Aggregate). This at least shows that Zeno was interested in the παρέγκλισις. Whether he connected that interest with his criticism of Euclidean geometry is, of course, another matter.

mensurability (συμμετρία). Although this argument is interesting, it is—as one would expect—ultimately unsuccessful. In general, since Euclidean geometry is consistent,[15] any 'strong' (*i.e.*, *reductio*) dialectical argument directed against it will either be invalid or will beg the question by tacitly *assuming* the existence of indivisible quanta of magnitude. There are also 'weaker' forms of dialectical argument, however. The existence of indivisible quanta is indeed assumed. And this assumption is 'applied' to geometrical theory, as it had developed in antiquity, in an attempt to work out the consequences of this very un-geometrical assumption. I have presented one possible illustration of such dialectical reasoning with respect to a criticism of the parallel postulate and the Epicurean doctrine of the παρέγκλισις.

ARIZONA STATE UNIVERSITY AND THE NATIONAL HUMANITIES CENTER
August, 1989

[15] Actually, only a slightly weaker claim has been proved: Euclidean geometry is consistent *if* elementary number theory is consistent.